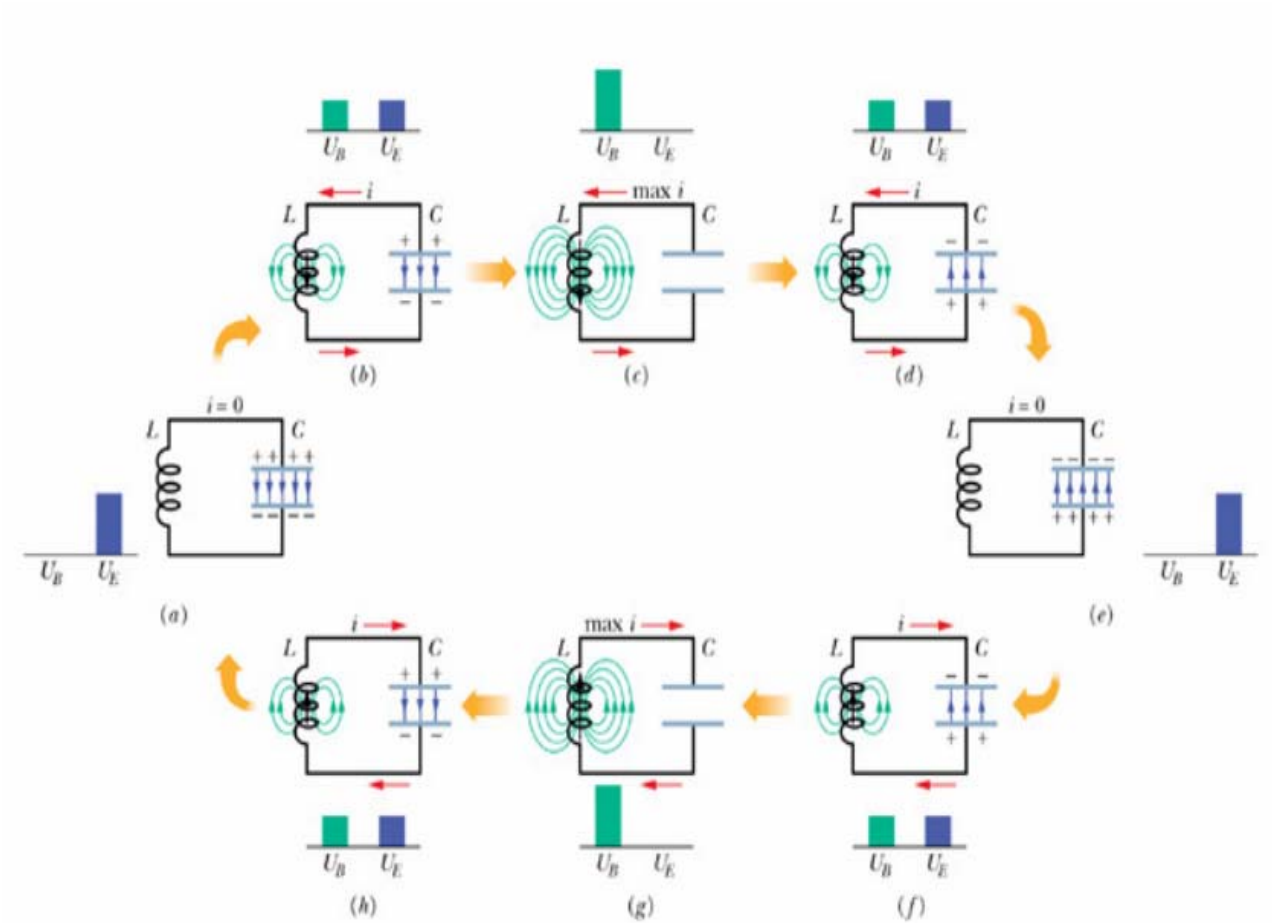
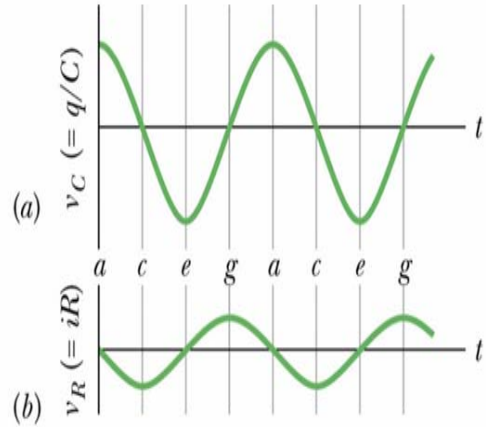
Oscillazioni e onde

- Circuiti oscillanti
- Onde
- Onde elettromagnetiche

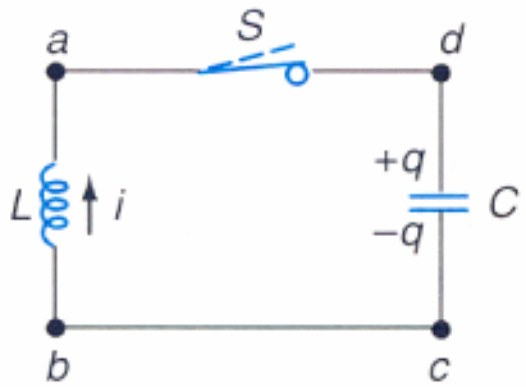
Citcuito LC





Circuito LC

$$(V_b - V_a) + (V_c - V_b) + (V_d - V_c) + (V_a - V_d) = 0$$



$$L\frac{di}{dt} + \frac{q}{C} = 0 \qquad i = \frac{dq}{dt} \qquad \frac{di}{dt} = \frac{d^2q}{dt^2}$$

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

Eq. dell'oscillatore armonico

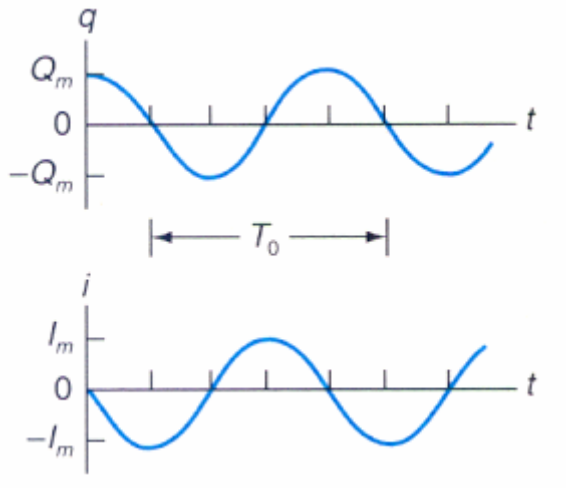
$$q(t) = Q_m \cos(\omega_0 t + \varphi) \qquad i(t) = \frac{dq}{dt} = -\omega_0 Q_m \sin(\omega_0 t + \varphi)$$

$$\frac{d^2q}{dt^2} = -\omega_0^2 Q_m \cos(\omega_0 t + \varphi) \qquad i_m = \omega_0 Q_m$$

$$-\omega_0^2 Q_m \cos(\omega_0 t + \varphi) = -\frac{1}{LC} Q_m \cos(\omega_0 t + \varphi)$$

Circuito LC

$$\omega_0^2 = \frac{1}{LC} \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$



condizioni iniziali $q = Q_0, i = 0$

$$q = Q_0 \cos(\omega_0 t) \qquad i = -i_m \sin(\omega_0 t)$$

$$U = U_E + U_B \qquad U_E = \frac{q^2}{2C} = \frac{Q_m^2}{2C}\cos^2(\omega_0 t + \varphi)$$

Energia

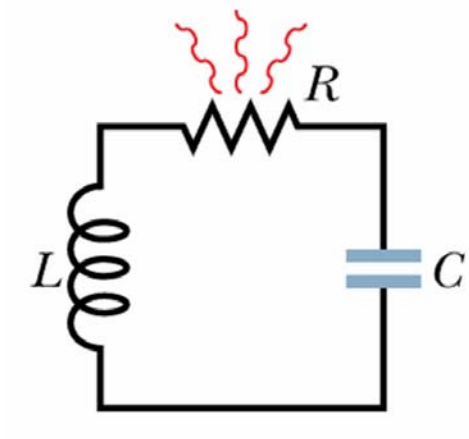
$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}Li_m^2 \sin^2(\omega_0 t + \varphi)$$

$$\frac{1}{2}Li_m^2 = \frac{1}{2}L(\omega_0 Q_m)^2 = \frac{1}{2}LQ_m{}^2\frac{1}{LC} = \frac{Q_m^2}{2C}$$

$$\boxed{U = \frac{1}{2}Li_m^2 = \frac{Q_m^2}{2C}}$$

Circuito RLC

$$f(t) = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt.$$



$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = \frac{df}{dt}$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

$$i(t) = e^{kt}$$

$$e^{kt}\left[Lk^2 + Rk + \frac{1}{C}\right] = 0.$$

$$Lk^2 + Rk + \frac{1}{C} = 0$$

Circuito RLC

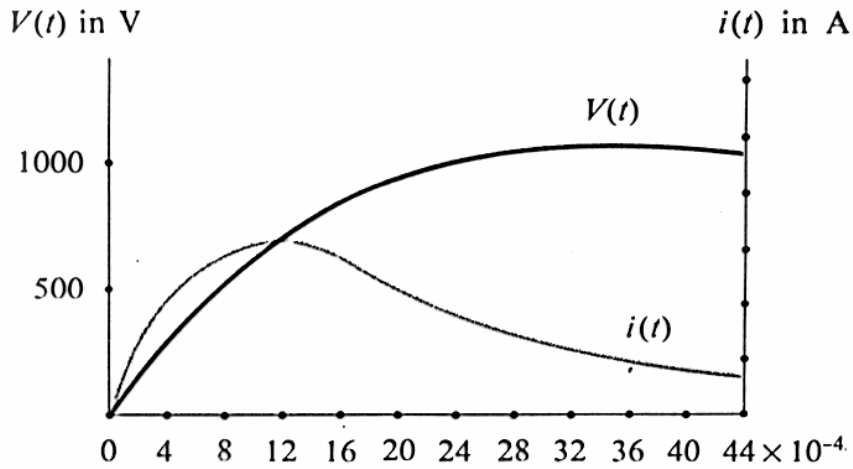
$$k_1,\ k_2 = \frac{-R \pm \sqrt{R^2 - 4\dfrac{L}{C}}}{2L}$$

$$i(t) = A_1 e^{k_1 t} + A_2 e^{k_2 t}.$$

$$R^2 \cong 4\frac{L}{C}$$

$$R^2 > 4\frac{L}{C}$$

$$R^2 = 4\frac{L}{C}$$

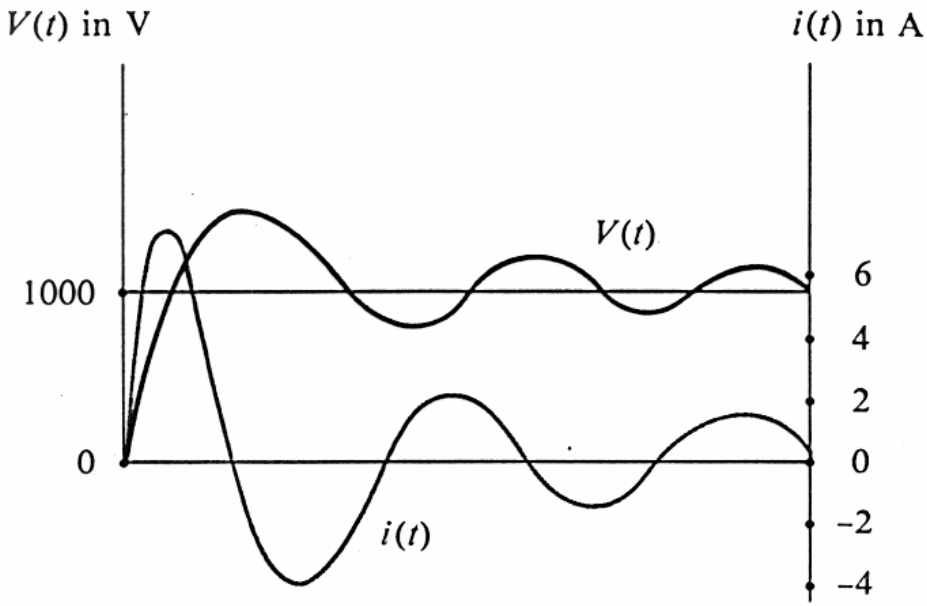


Circuito RLC

$$R^2 < 4\frac{L}{C}$$

$$i(t) = e^{-\frac{R}{2L}t}\left(A_1 e^{i\omega_0 t} + A_2 e^{-i\omega_0 t}\right)$$

$$\nu_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}.$$



$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

Circuito RLC