# Equazioni di Maxwell

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{\sum q}{\varepsilon_0}$$

(Legge di Gauss)

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

(Legge di Gauss per il campo magnetico)

$$\oint \mathbf{E} \cdot dl = -\frac{d}{dt}\int \mathbf{B} \cdot d\mathbf{S}$$

(Legge di Faraday-Neumann-Lenz)

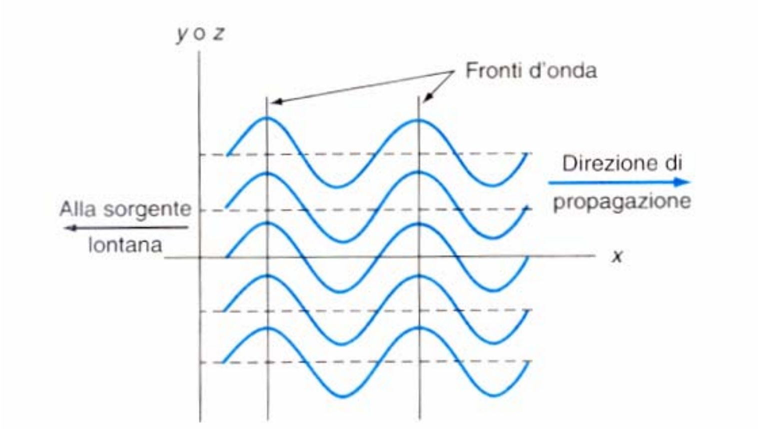
$$\oint \mathbf{B} \cdot dl = \mu_0 \sum I + \varepsilon_0 \mu_0 \frac{d}{dt}\int \mathbf{E} \cdot d\mathbf{S}$$

(Legge di Ampere-Maxwell)

# Onde piane, E ⊥ B

$$\mathbf{E} = \mathbf{E}(x,t)$$

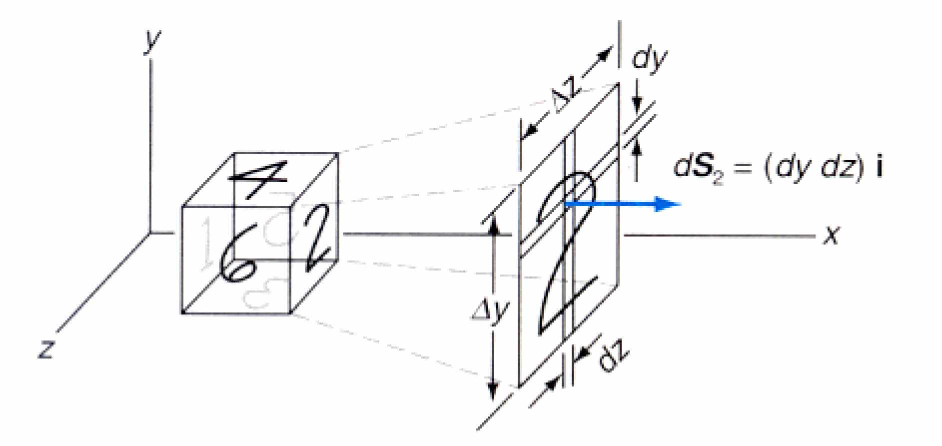
$$\mathbf{B} = \mathbf{B}(x,t)$$




$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{\sum q}{\varepsilon_0} = 0$$

$$\int \mathbf{E}(1) \cdot d\mathbf{S}_1 + \int \mathbf{E}(2) \cdot d\mathbf{S}_2 + \int \mathbf{E}(3) \cdot d\mathbf{S}_3 + \int \mathbf{E}(4) \cdot d\mathbf{S}_4 + \int \mathbf{E}(5) \cdot d\mathbf{S}_5 + \int \mathbf{E}(6) \cdot d\mathbf{S}_6 = 0$$

$$d\mathbf{S}_1 = -(dydz)\mathbf{i} \text{ ecc.}$$



# Onde piane trasversali, E ⊥ B

$$-\int E_x(1)dydz + \int E_x(2)dydz - \int E_y(3)dxdz + \int E_y(4)dxdz - \int E_z(5)dxdy + \int E_z(6)dxdy = 0$$

$$E_y(3) = E_y(4)$$

$$E_z(5) = E_z(6)$$

$$\Rightarrow$$

$$-\int E_x(1)dydz + \int E_x(2)dydz = 0$$

$$-E_x(1)\int dydz + E_x(2)\int dydz = 0$$

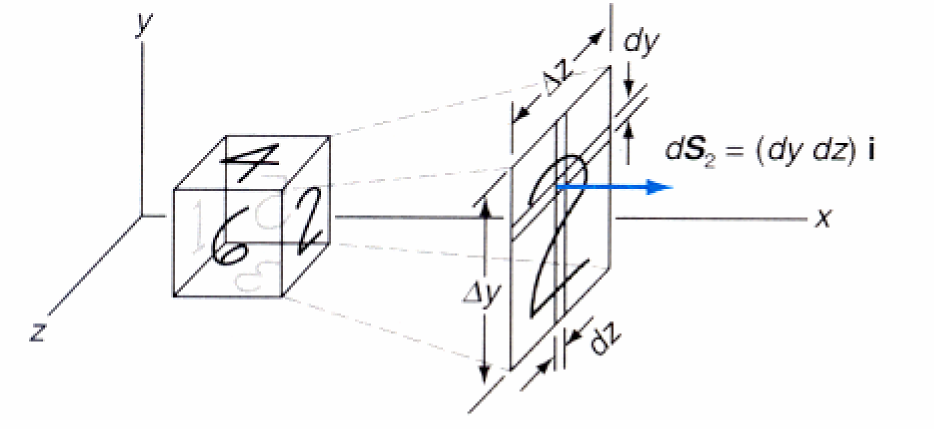
$$E_x(1)\Delta y\Delta z = E_x(2)\Delta y\Delta z$$

$E_x$ e' indipendente da $x$

$$E_x = 0$$

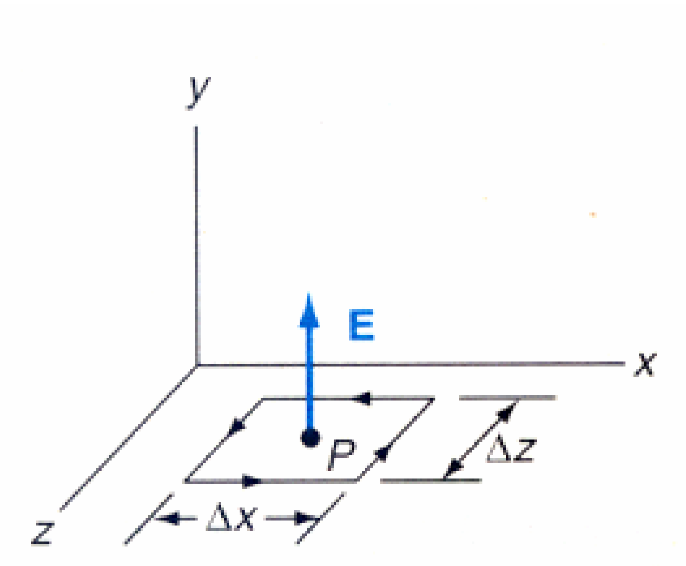
analogamente

$$B_x = 0$$



# E ⊥ B

$$\mathbf{E} = E_y(x,t)\,\mathbf{j}$$




$$\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{S}$$

$$= 0$$

$$\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{S} = 0 \qquad d\mathbf{S} = (dxdz)\,\mathbf{j}$$

$$\int \mathbf{B}\cdot d\mathbf{S} \approx B_y(\Delta x \Delta z)$$

$$0 \approx \frac{d}{dt}\left[B_y(\Delta x \Delta z)\right] = (\Delta x \Delta z)\frac{\partial}{\partial t}B_y$$

$$\boxed{B_y = 0}$$

$$\boxed{\mathbf{B} = B_z(x,t)\,\mathbf{k}} \qquad \mathbf{E} = E_y(x,t)\,\mathbf{j} \qquad \boxed{\mathbf{B} \perp \mathbf{E}}$$

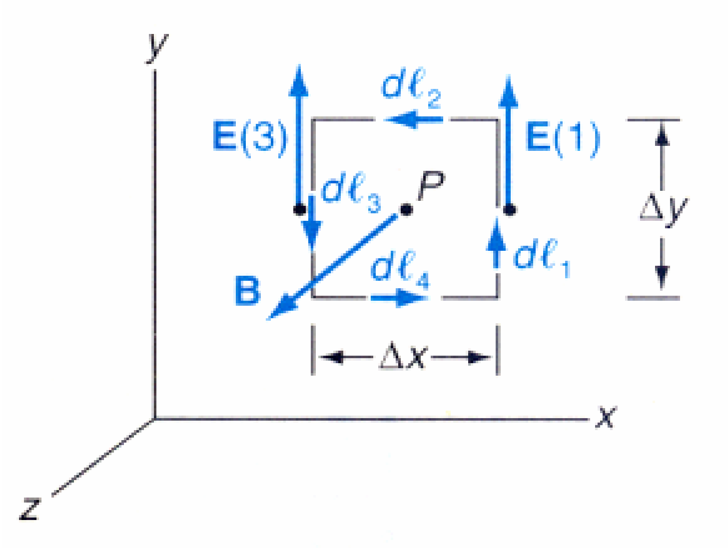
Onda polarizzata

# Equazione delle onde

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{S}$$

$$\oint \mathbf{E}\cdot d\mathbf{l} = \int \mathbf{E}(1)\cdot dl_1 + \int \mathbf{E}(2)\cdot dl_2 + \int \mathbf{E}(3)\cdot dl_3 + \int \mathbf{E}(4)\cdot dl_4$$

$$\int \mathbf{E}(2)\cdot dl_2 = \int \mathbf{E}(4)\cdot dl_4 = 0$$




$$\oint \mathbf{E}\cdot d\mathbf{l} = \int \mathbf{E}(1)\cdot dl_1 + \int \mathbf{E}(3)\cdot dl_3$$

$$= \left[E_y(1) - E_y(3)\right]\Delta y$$

$$E_y(1) - E_y(3) = \frac{E_y(1) - E_y(3)}{\Delta x}\Delta x \approx \frac{\partial E_y}{\partial x}\Delta x$$

$$\oint \mathbf{E}\cdot d\mathbf{l} = \frac{\partial E_y}{\partial x}\Delta x \Delta y$$

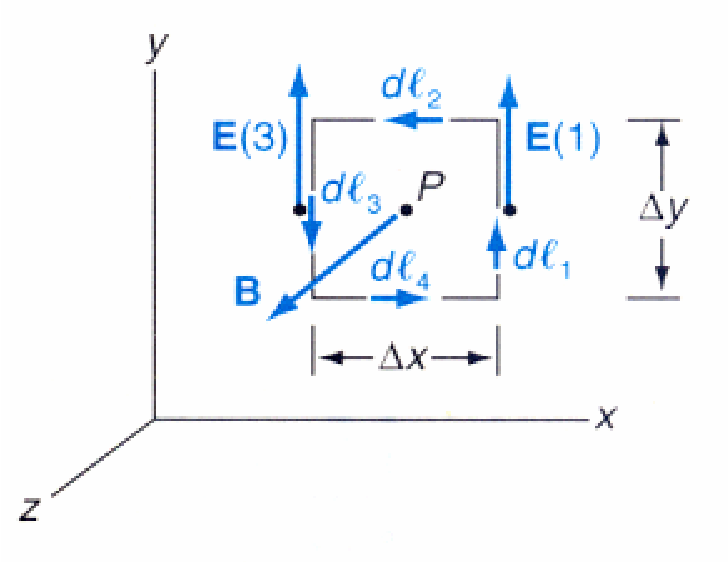
# Equazione delle onde

$$-\frac{d}{dt}\int \mathbf{B}\cdot d\mathbf{S} \qquad d\mathbf{S} = (dxdy)\mathbf{k}$$

$$\int \mathbf{B}\cdot d\mathbf{S} \approx B_z(\Delta x \Delta y)$$

$$\frac{\partial E_y}{\partial x}\Delta x \Delta y = -\frac{\partial B_z}{\partial t}\Delta x \Delta y$$

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$



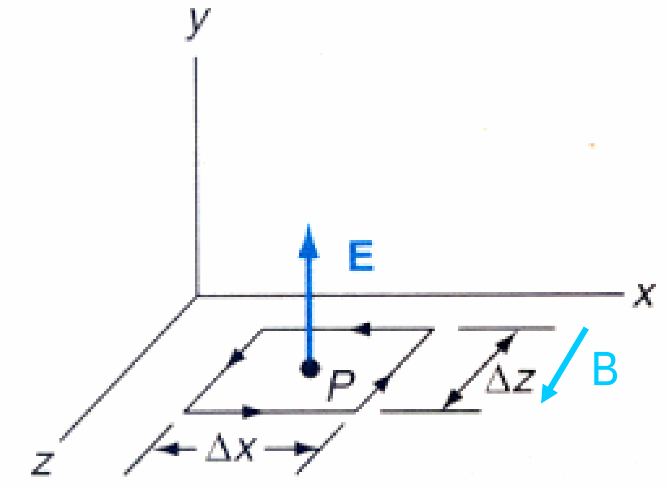
# Equazione delle onde

$$\oint \mathbf{B}\cdot d\mathbf{l} = \varepsilon_0\mu_0 \frac{d}{dt}\int \mathbf{E}\cdot d\mathbf{S}$$

$$\oint \mathbf{B}\cdot d\mathbf{l} = \left[B_z(1) - B_z(3)\right]\Delta z$$

$$B_z(3) - B_z(1) \approx \frac{\partial B_z}{\partial x}\Delta x$$

$$\oint \mathbf{B}\cdot d\mathbf{l} = -\frac{\partial B_z}{\partial x}\Delta x\Delta z$$




$$\frac{d}{dt}\int \mathbf{E}\cdot d\mathbf{S} = \frac{\partial E_y}{\partial t}\Delta x\Delta z$$

$$\boxed{\frac{\partial B_z}{\partial x} = -\varepsilon_0\mu_0 \frac{\partial E_y}{\partial t}}$$

# Equazione delle onde

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad \frac{\partial B_z}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t}$$

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial}{\partial x}\frac{\partial B_z}{\partial t} \qquad \frac{\partial}{\partial t}\frac{\partial B_z}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 B_z}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.00 \cdot 10^8 \, m/s$$

# Onde elettromagnetiche

$$E_y = E_0 \sin\left(k_e x - \omega_e t\right)$$

$$B_z = B_0 \sin\left(k_b x - \omega_b t + \varphi\right)$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{\omega_e}{k_e} = \frac{\omega_b}{k_b}$$

$$\frac{\partial E_y}{\partial x} = k_e E_0 \cos\left(k_e x - \omega_e t\right)$$

$$\frac{\partial B_z}{\partial t} = -\omega_b B_0 \cos\left(k_b x - \omega_b t + \varphi\right) = -k_b c B_0 \cos\left(k_b x - \omega_b t + \varphi\right)$$

$$\omega = ck$$

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}} \Rightarrow k_e E_0 \cos\left(k_e x - \omega_e t\right) = k_b c B_0 \cos\left(k_b x - \omega_b t + \varphi\right)$$

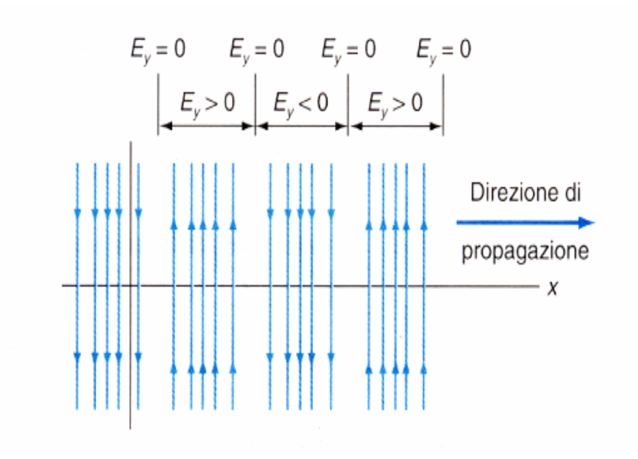
$$k_e = k_b \qquad \omega_e = \omega_b \qquad \varphi = 0$$

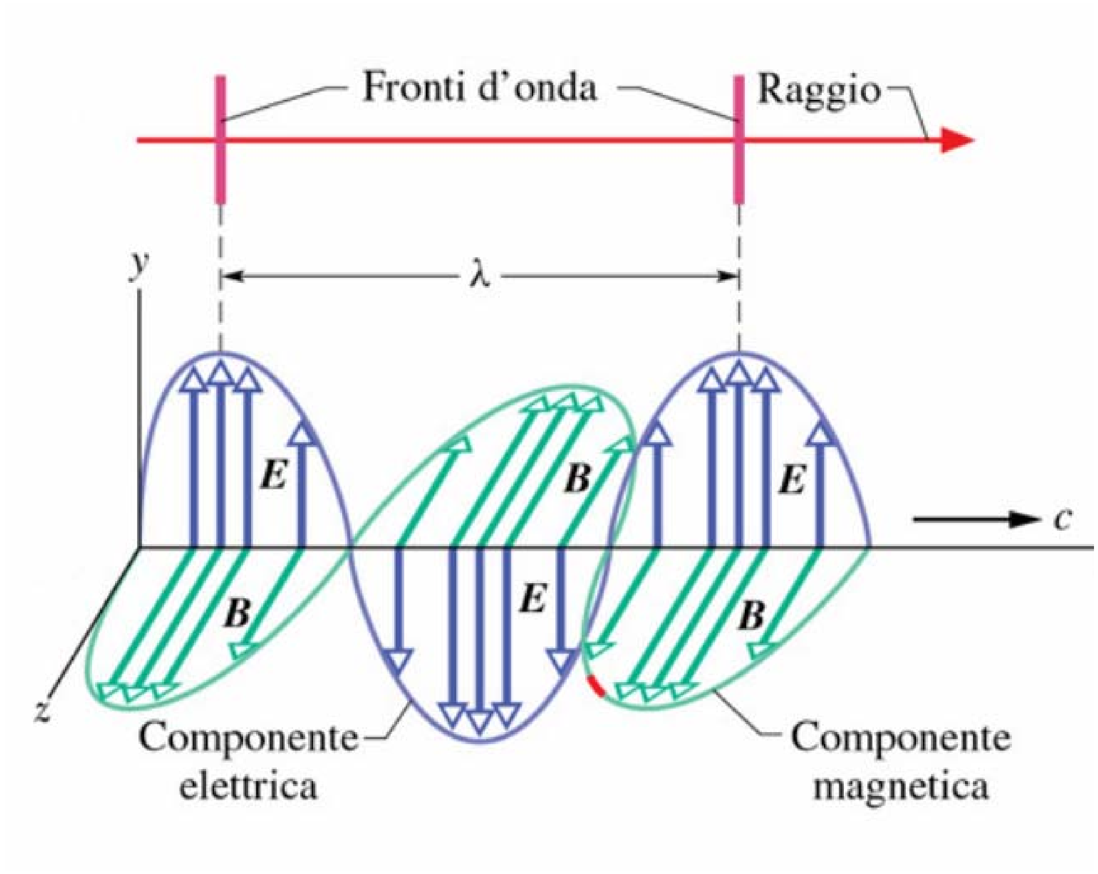
# Onde elettromagnetiche

$$kE_0 \cos(kx - \omega t) = kcB_0 \cos(kx - \omega t) \quad \Rightarrow \quad E_0 = cB_0$$

$$E_y = E_0 \sin(kx - \omega t)$$

$$B_z = B_0 \sin(kx - \omega t)$$



# Intensita' delle onde elettromagnetiche

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad u_B = \frac{1}{2\mu_0}B^2 \qquad E_y = cB_z \qquad \varepsilon_0 = \frac{1}{c^2\mu_0}$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 E_y^2 = \frac{1}{2}\frac{1}{\mu_0 c^2}(cB_z)^2 = \frac{\frac{1}{2}B^2}{\mu_0} = u_B$$

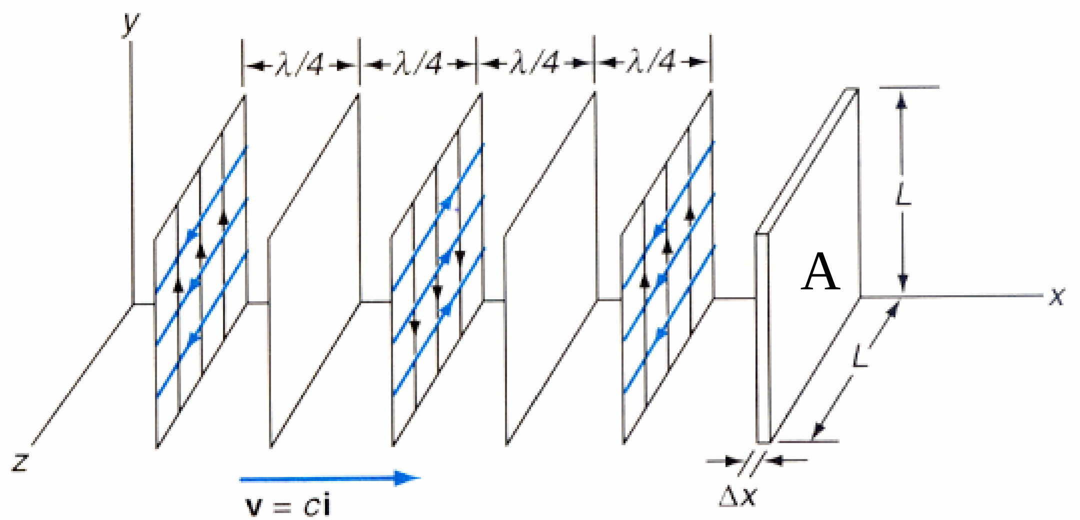
$u = u_E + u_B$ densita' di energia e.m.

$$u = \varepsilon_0 E^2$$

$$\Delta U = u(A\Delta x)$$

$$\Delta t = \frac{\Delta x}{c}$$

$$\frac{\Delta U}{\Delta t} = uA\frac{\Delta x}{\Delta t} = uAc$$



# Intensita' delle onde elettromagnetiche

$$S = \frac{1}{A}\frac{\Delta U}{\Delta t} = uc \qquad \text{intensita'}$$

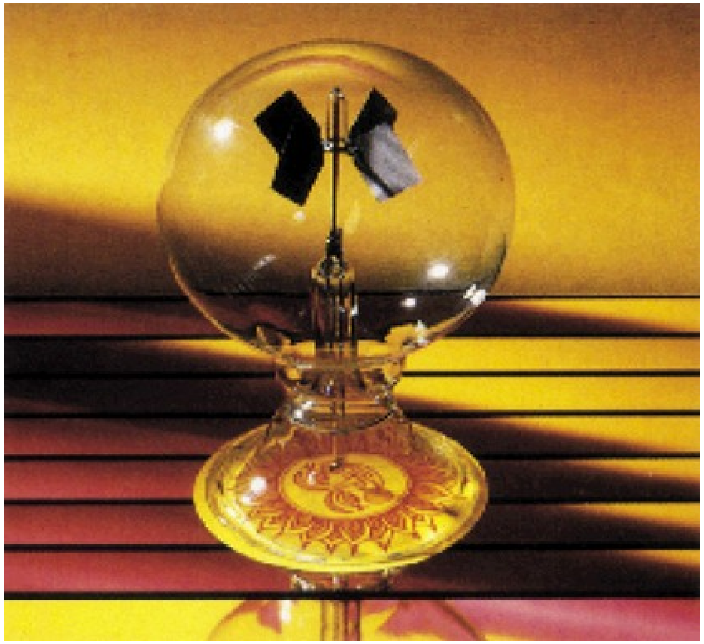
$$S = \varepsilon_0 E^2 c = \varepsilon_0 (cB) E c = \varepsilon_0 \left( \frac{1}{\varepsilon_0 \mu_0} \right) EB$$

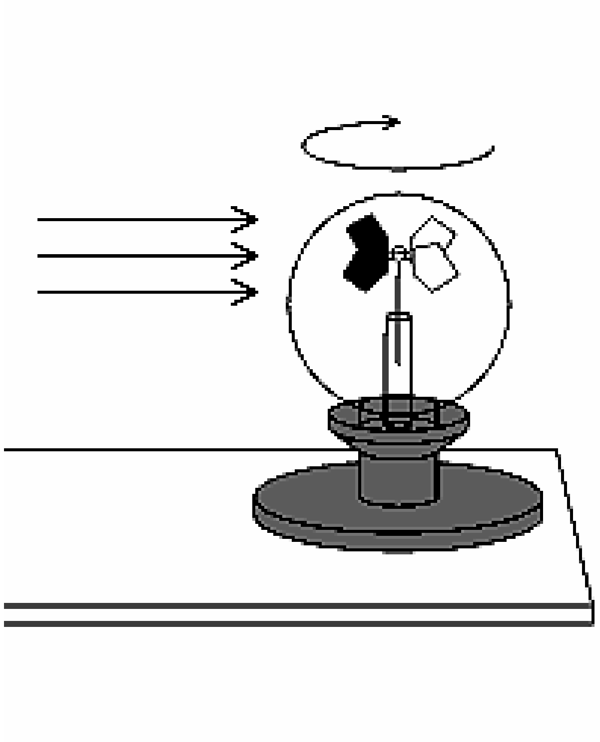
$$\mathbf{E} \wedge \mathbf{B} = E_y B_z \mathbf{i} \quad \Rightarrow \quad \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \wedge \mathbf{B} \quad \text{vettore di Poynting}$$

$$\mathrm{S} = \frac{1}{\mu_0} E_0 B_0 \sin^2 (kx - \omega t)$$

$$\overline{S} = \frac{1}{2\mu_0} E_0 B_0 = \frac{1}{2} \varepsilon_0 E_0^2 c = \frac{c B_0^2}{2\mu_0}$$

# Pressione di radiazione: Radiometro

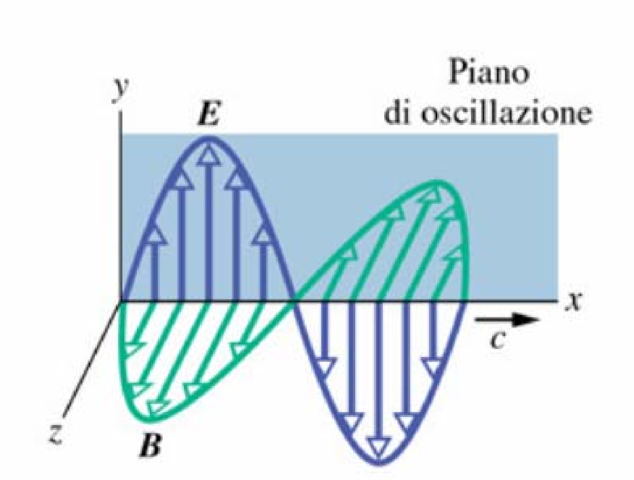


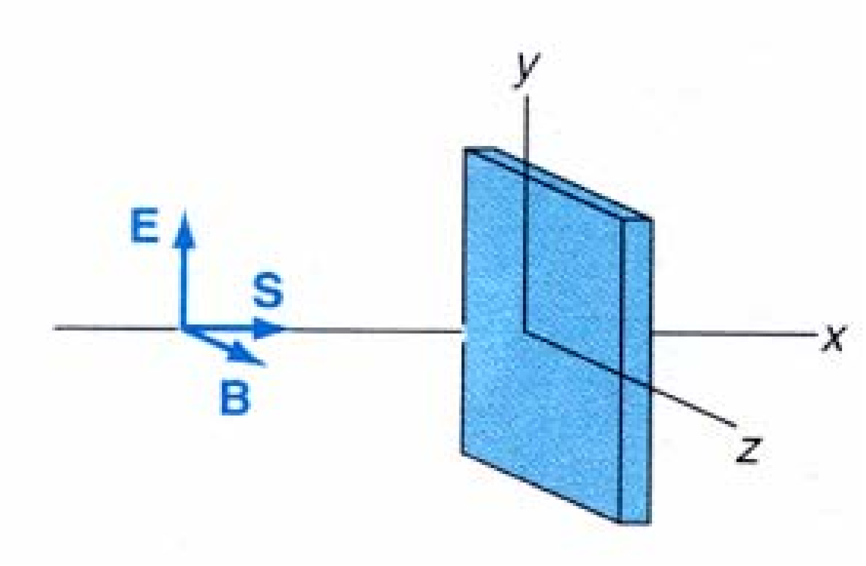


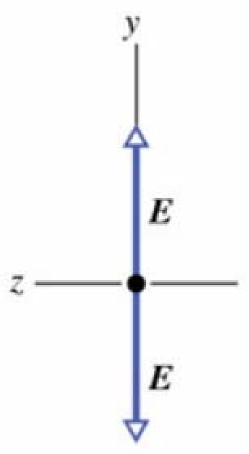
# Pressione di radiazione

$$p = \frac{S}{c} \qquad (assorbimento)$$

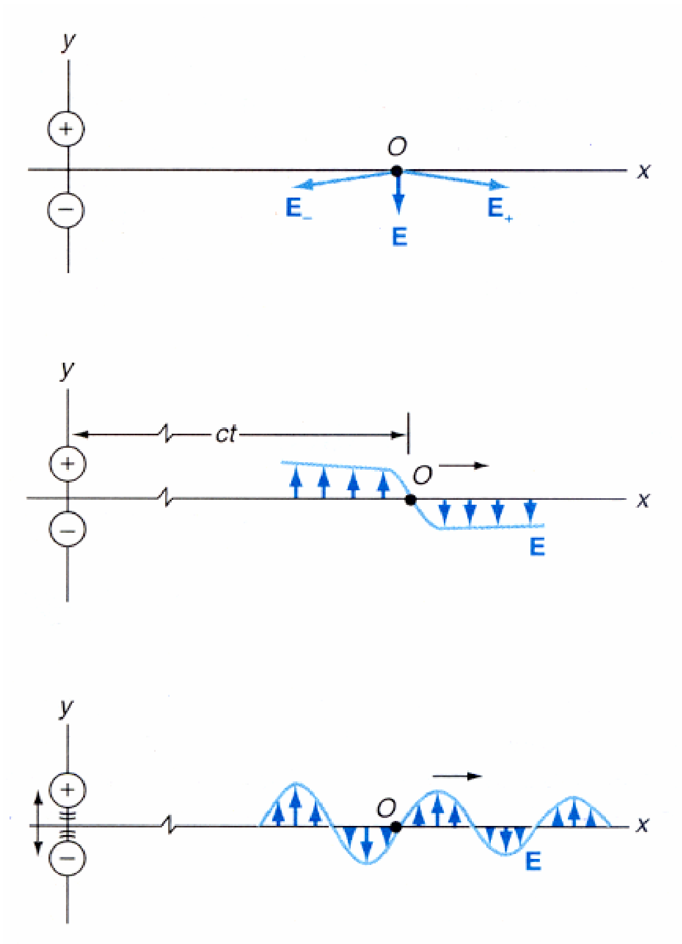
$$p = \frac{2S}{c} \qquad (riflessione)$$

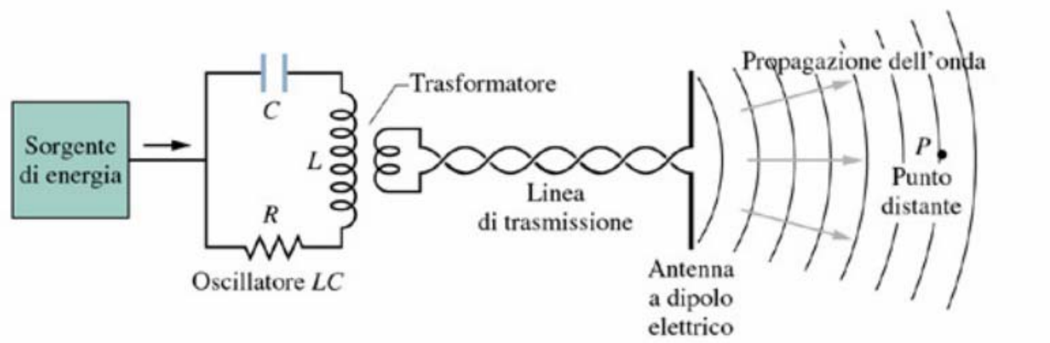


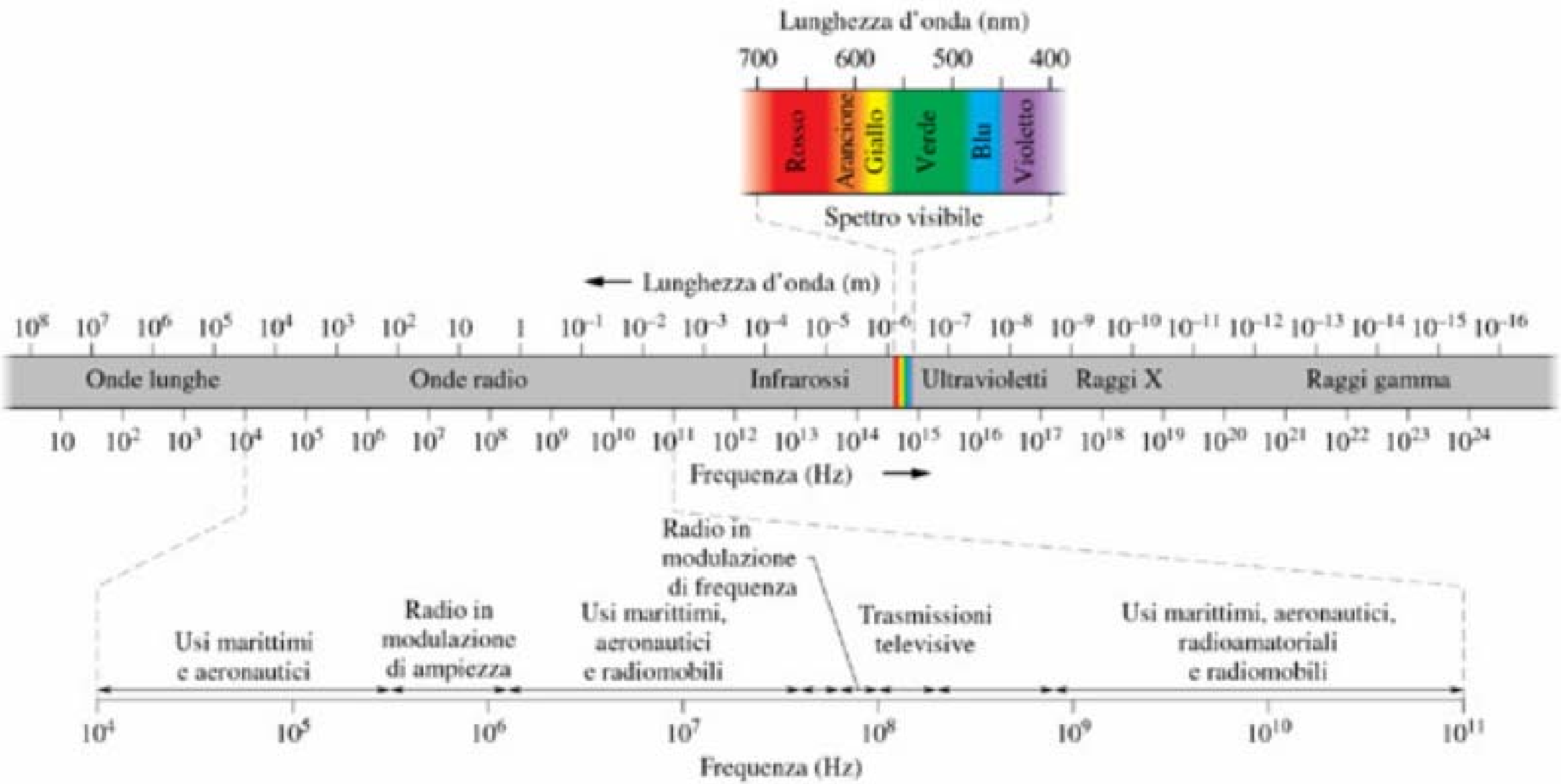



# Emissione di onde e.m.



# Spettro elettromagnetico




$$\lambda = \frac{c}{\nu}$$

# Spettro visibile