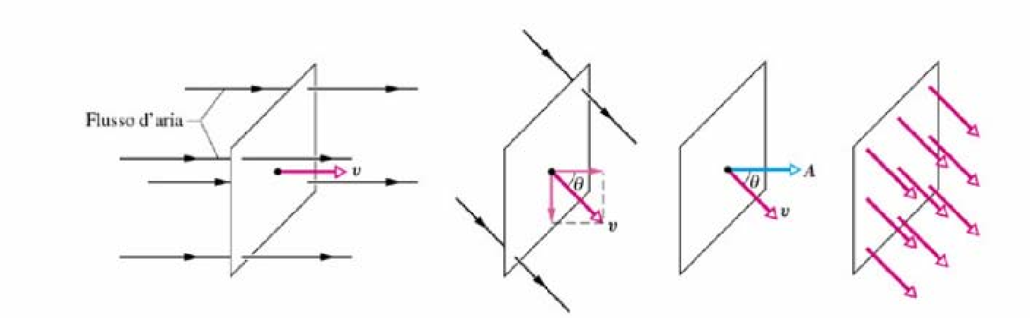
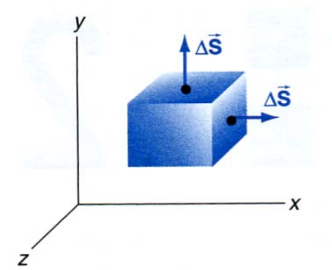
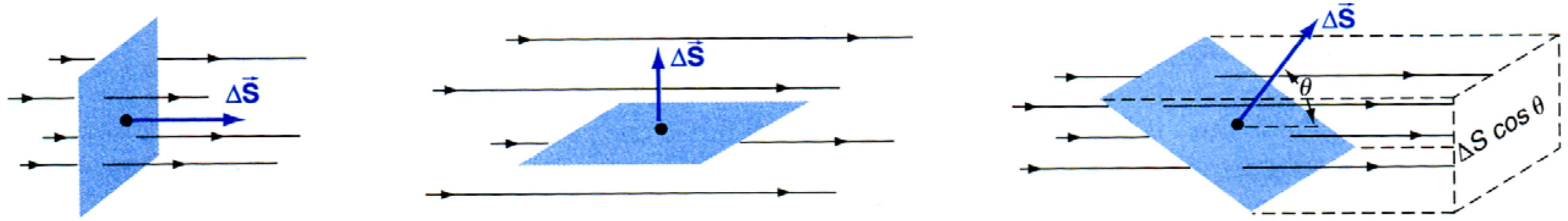
Legge di Gauss

- Flusso
- Legge di Gauss e Legge di Coulomb
- Uso della L. di Gauss per determinare E
- Proprietà dei conduttori

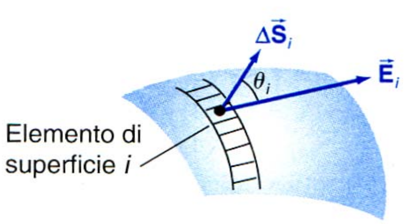
Flusso







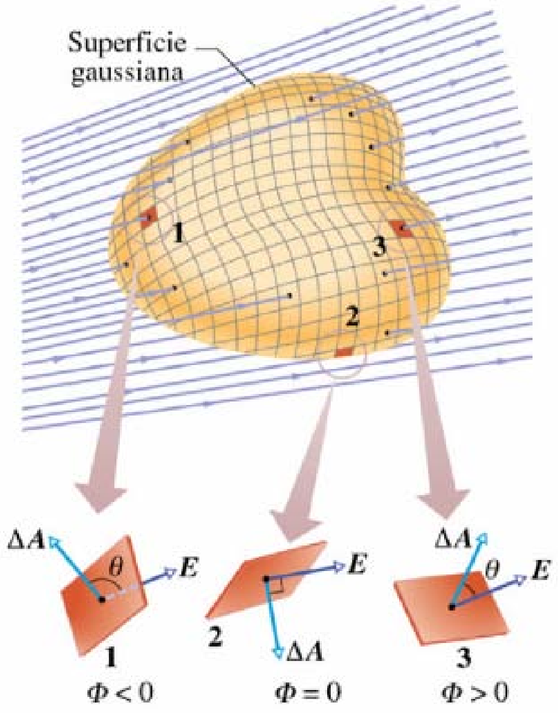
$$\Phi_E = \vec{\mathbf{E}} \cdot \Delta\vec{\mathbf{S}} = E\Delta S \cos\theta \quad \text{Flusso } [\mathrm{Nm^2/C}]$$



$$\Phi_E = \lim_{\Delta_{S_i} \to 0} \sum_i \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{S}}_i = \iint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}}$$

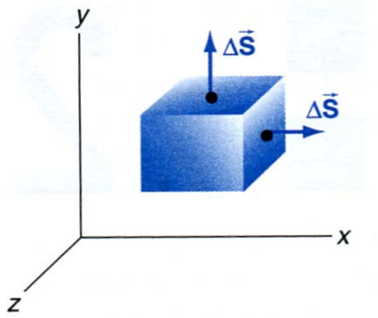
Flusso

$$\Phi_E = \iint\limits_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \iint\limits_S \vec{\mathbf{E}} \cdot \hat{\mathbf{n}} dS = \iint\limits_S E \cos\theta dS$$



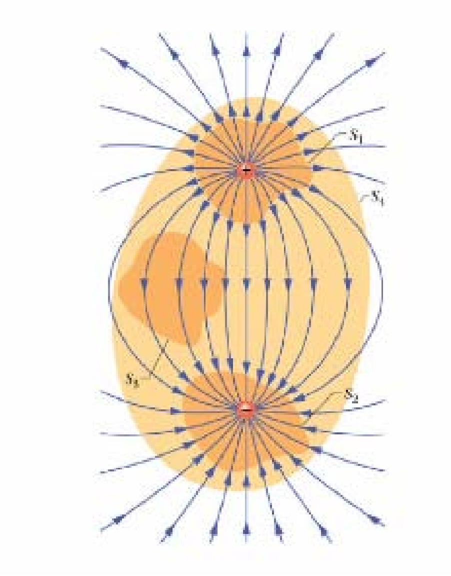
Se la superficie è chiusa:

$$\Phi_E = \oiint\limits_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}}$$



Legge di Gauss

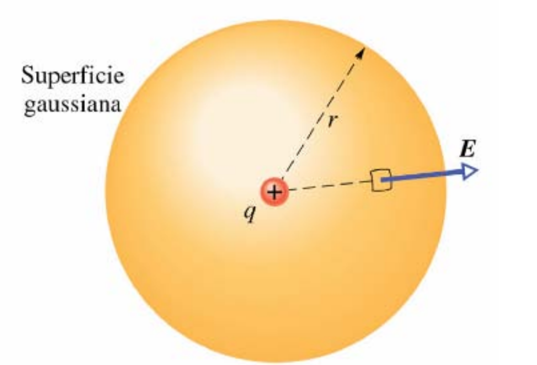
$$\Phi_E = \frac{Q_{\text{int}}}{\varepsilon_0} \qquad \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{1}{\varepsilon_0} \iiint_{V_{\text{int}}} \rho \, dv$$



L. di Gauss

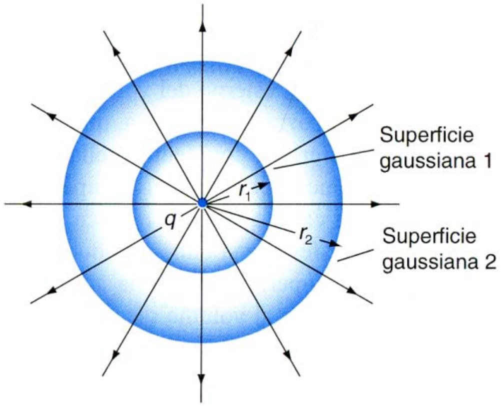
$$\Phi_E = \oiint_{Sfera} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \oiint_{Sfera} E_r \, dS = E_r \oiint_{Sfera} dS = E_r (4\pi r^2)$$

$$\vec{\mathbf{E}} = \frac{q}{4\pi\varepsilon_0 r^2} \hat{\mathbf{r}}$$

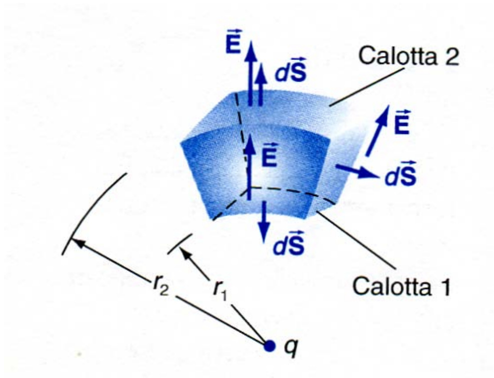


L. di Gauss dalla L. di Coulomb

$$\vec{\mathbf{E}} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \qquad E(r_{1,2})[4\pi r_{1,2}{}^2] = \frac{q}{\varepsilon_0}$$



Carica esterna alla superficie:

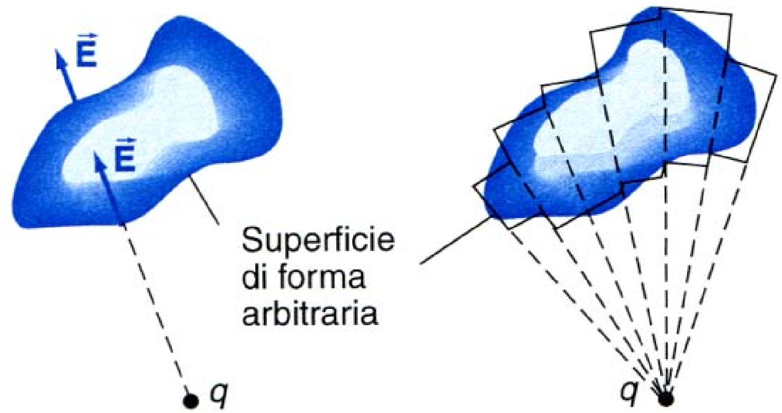


$$\Phi_1 = \oiint_{S_1} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \iint_{S_1} -E dS = \iint_{S_1} -\frac{q}{4\pi\varepsilon_0 r_1^2} dS =$$

$$= -\frac{q}{4\pi\varepsilon_0 r_1^2}\iint_{S_1} dS = -\frac{q}{4\pi\varepsilon_0 r_1^2}\Delta S_1$$

$$\Phi_2 = \frac{q}{4\pi\varepsilon_0 r_2^2}\Delta S_2 \qquad \frac{\Delta S_1}{\Delta S_2} = \frac{r_1^2}{r_2^2} \qquad \Phi_2 = -\Phi_1 \qquad \Phi_E = 0$$

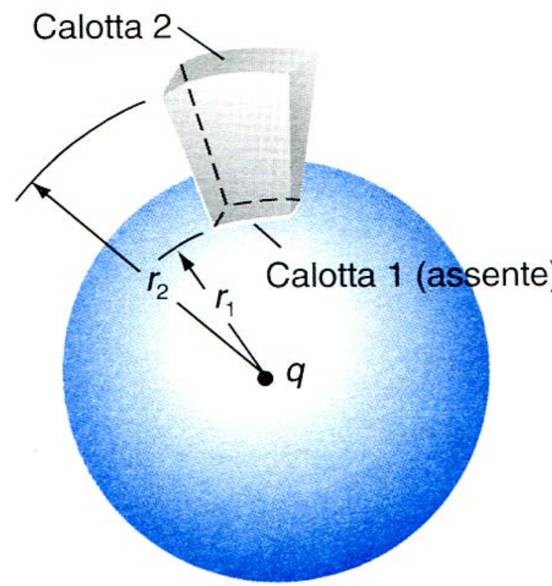
L. di Gauss dalla L. di Coulomb

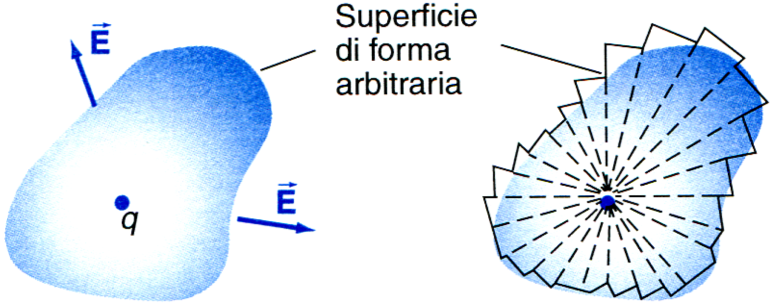


Carica interna alla superficie:

$$\Phi_E = \underset{Sfera}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \underset{Sfera}{\iint} E_r dS = E_r \underset{Sfera}{\iint} dS =$$

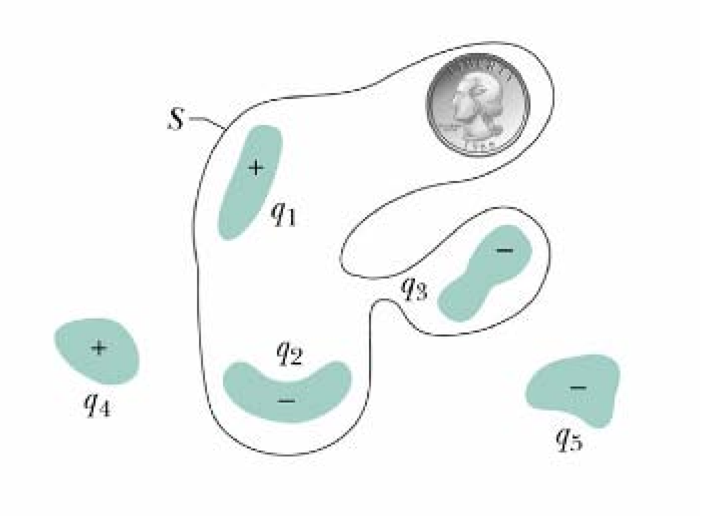
$$= \frac{q}{4\pi\varepsilon_0 r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}$$





L. di Gauss dalla L. di Coulomb

Cariche sia interne che esterne:



$$\Phi_E = \oiint_{S_{chiusa}} (\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 + \ldots) \cdot d\vec{\mathbf{S}} =$$

$$\Phi_E = \oiint_{S_{chiusa}} \vec{\mathbf{E}}_1 \cdot d\vec{\mathbf{S}} + \oiint_{S_{chiusa}} \vec{\mathbf{E}}_2 \cdot d\vec{\mathbf{S}} + \oiint_{S_{chiusa}} \vec{\mathbf{E}}_3 \cdot d\vec{\mathbf{S}} + \ldots =$$

$$\Phi_E = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \frac{q_3}{\varepsilon_0} + 0 + 0 + \ldots$$

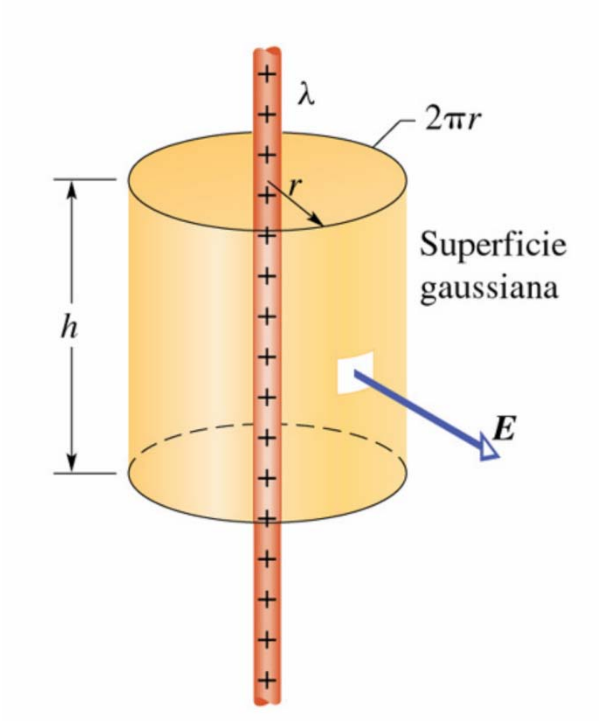
$$\Phi_E = \frac{Q_{\text{int}}}{\varepsilon_0}$$

Distribuzione lineare di cariche

$$\Phi_E = \oint\!\!\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = E_r (2\pi r h)$$

$$\Phi_E = \frac{Q_{\text{int}}}{\varepsilon_0} \qquad E_r 2\pi r h = \frac{\lambda h}{\varepsilon_0}$$

$$E_r = \frac{\lambda}{2\pi \varepsilon_0 r}$$

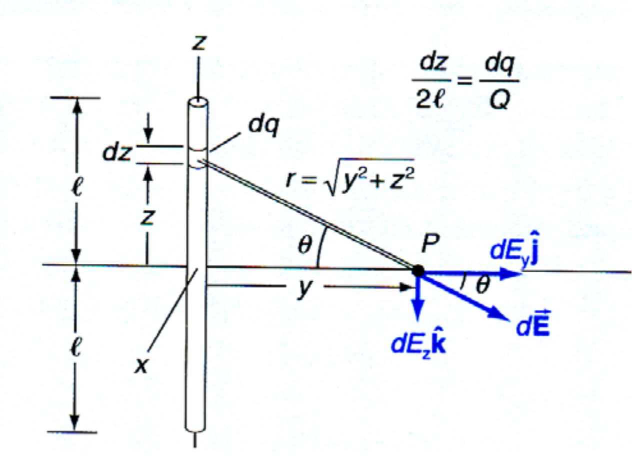


Campo elettrico

Distribuzione lineare:

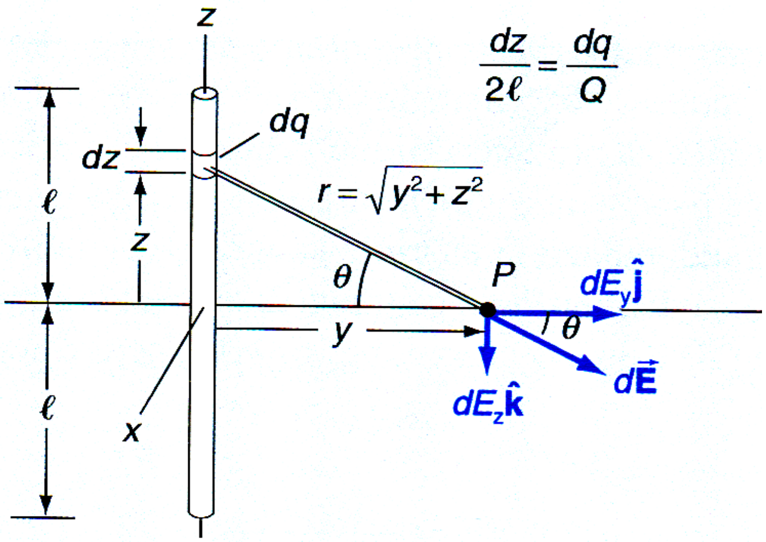
$$d\vec{\mathbf{E}} = dE_y\hat{\mathbf{j}} - dE_z\hat{\mathbf{k}} = (dEcos\theta)\hat{\mathbf{j}} - (dEsen\theta)\hat{\mathbf{k}}$$

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dz}{(y^2 + z^2)}\hat{\mathbf{r}}$$



Campo elettrico

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dz}{(y^2+z^2)}\hat{\mathbf{r}}$$



$$E_y = \int dE_y = \frac{\lambda y}{4\pi\varepsilon_0}\int_{-l}^{l}\frac{dz}{(y^2+z^2)^{3/2}}$$

$$\int_{-l}^{l}\frac{dz}{(y^2+z^2)^{3/2}} = \left.\frac{z}{y^2\sqrt{y^2+z^2}}\right|_{-l}^{+l} = \frac{2l}{y^2\sqrt{y^2+l^2}}$$

$$\cos\theta = \frac{y}{\sqrt{y^2+z^2}}$$

$$E_y = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{y}\frac{l}{\sqrt{l^2+y^2}}$$

se l>>y (R)

$$E \approx \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$$

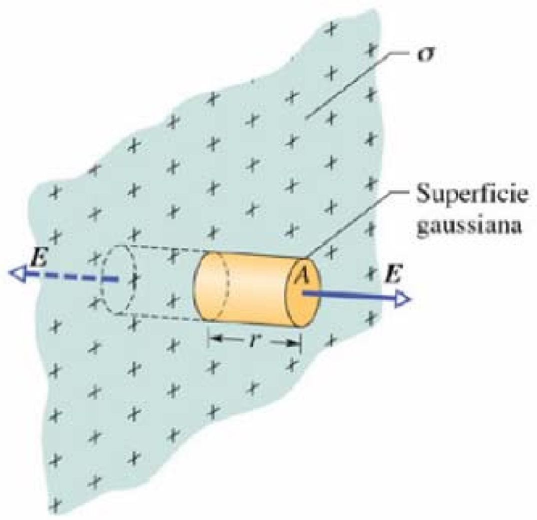
Distribuzione piana di cariche

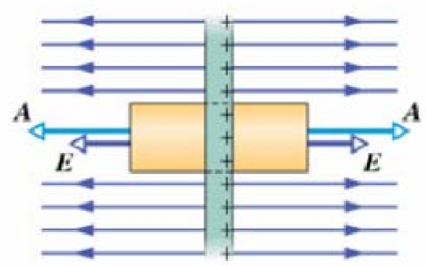
$$\Phi_E = \oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = E_r 2A$$

$$Q_{\text{int}} = \sigma A$$

$$2E_r A = \frac{\sigma A}{\varepsilon_0}$$

$$E_r = \frac{\sigma}{2\varepsilon_0}$$





Proprietà dei conduttori

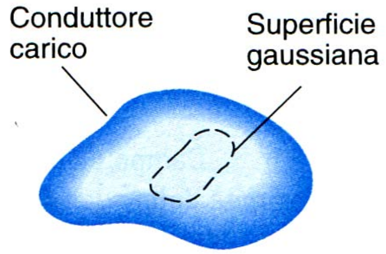
$E_{\text{int}} = 0$ (in condizioni statiche)

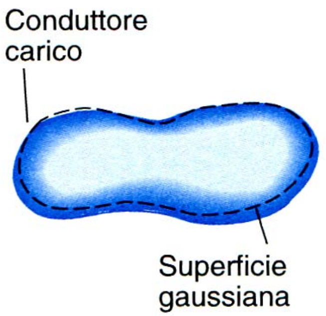
⇒ la carica è sulla superficie

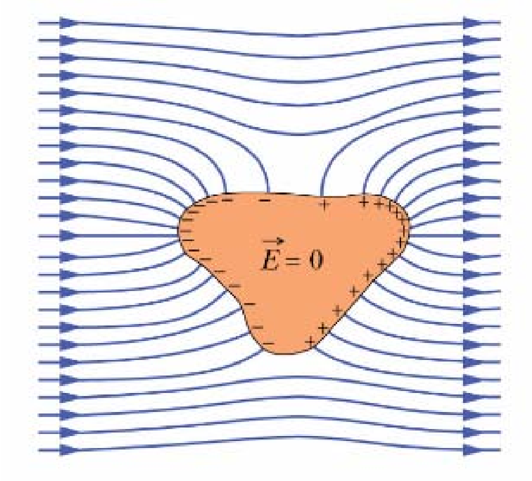
$\Phi_E = E_n \Delta S \qquad \sum q = \sigma \Delta S$

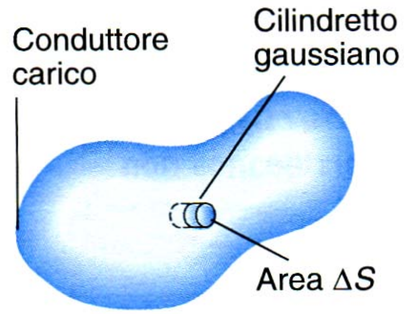
$$E_n \Delta S = \frac{\sigma \Delta S}{\varepsilon_0}$$

$$E_n = \frac{\sigma}{\varepsilon_0}$$









Gabbia di Faraday