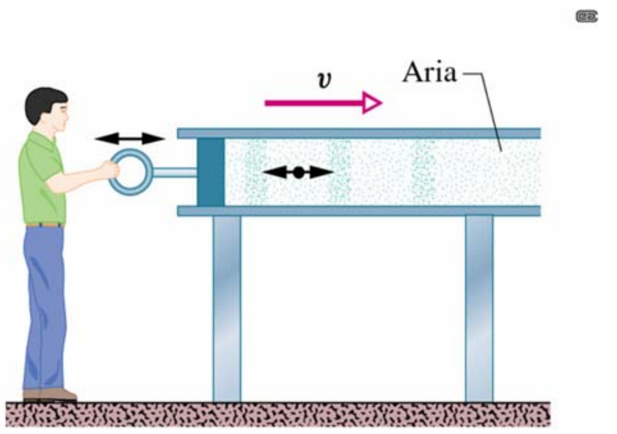
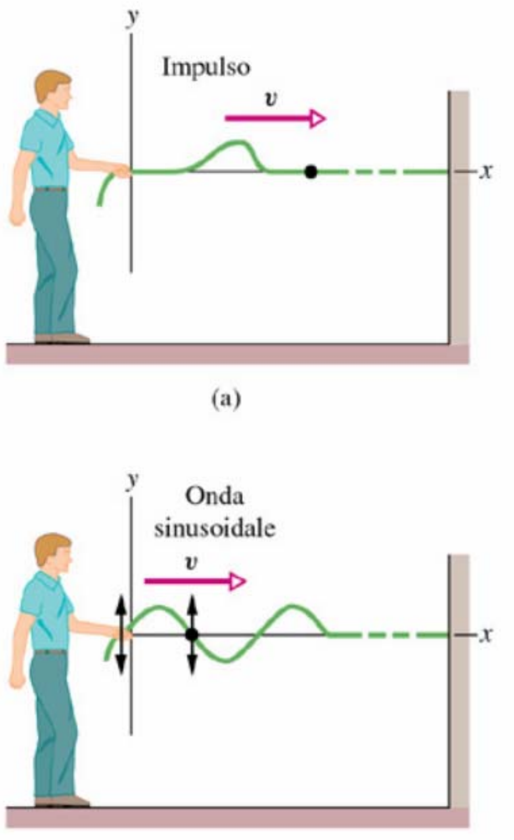
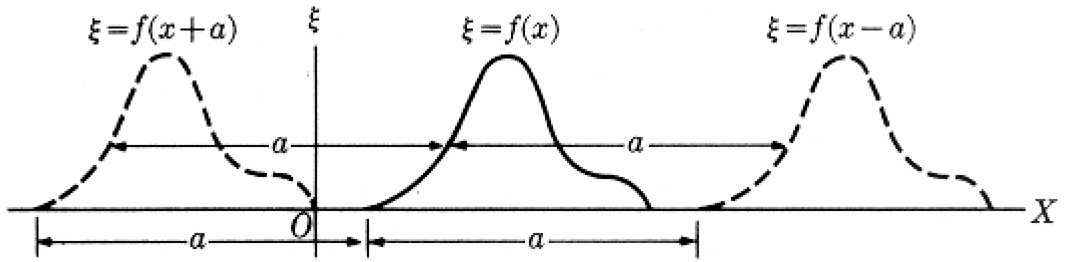
Le onde

Onde longitudinali ↔ trasversali



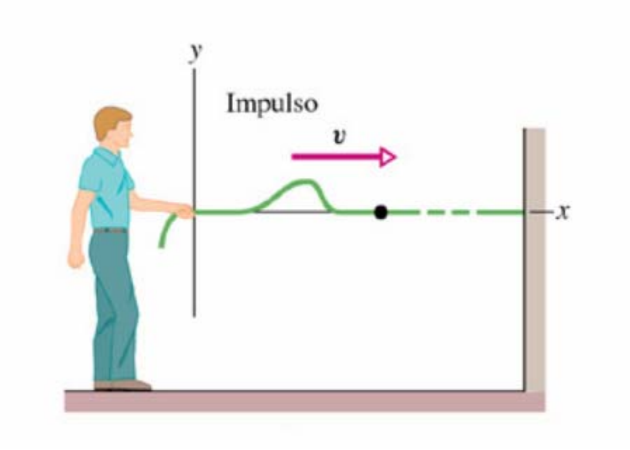




Onde progressive

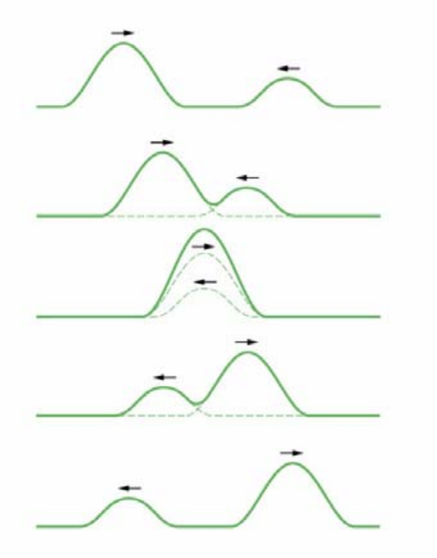
$$y(x,t) = f(x - vt)$$ Lungo +x

$$y(x,t) = f(x + vt)$$ Lungo -x



Es.

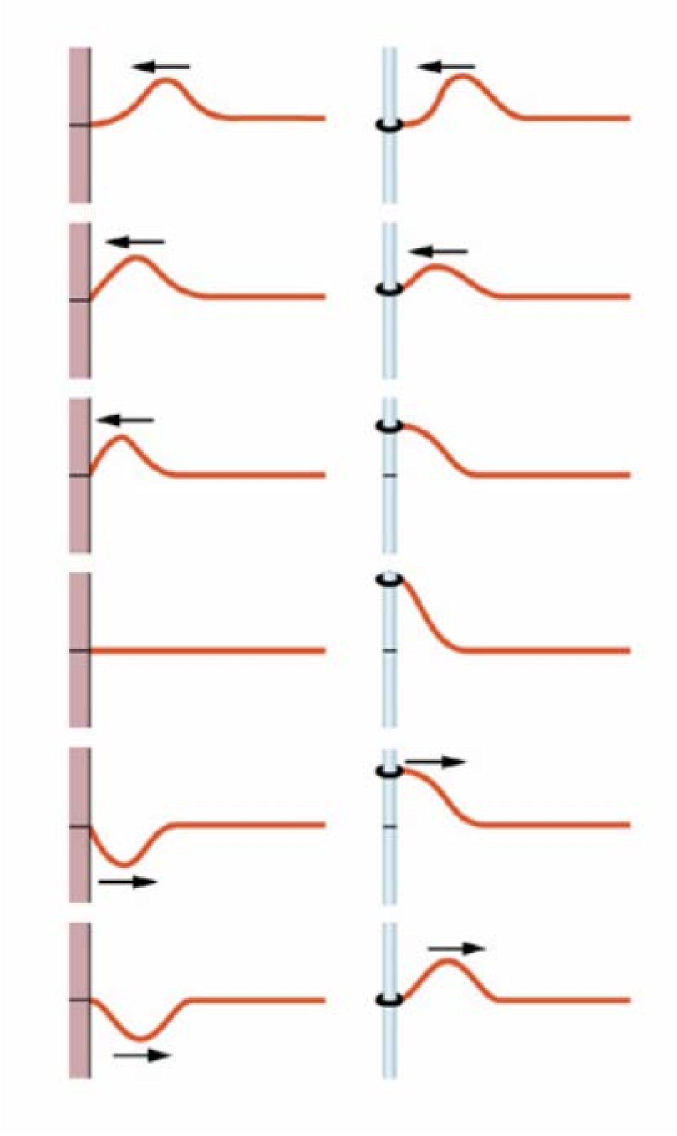
$$f(x) = \frac{y_0}{\left(\frac{x}{x_0}\right)^2 + 1} \qquad x \to x - vt$$

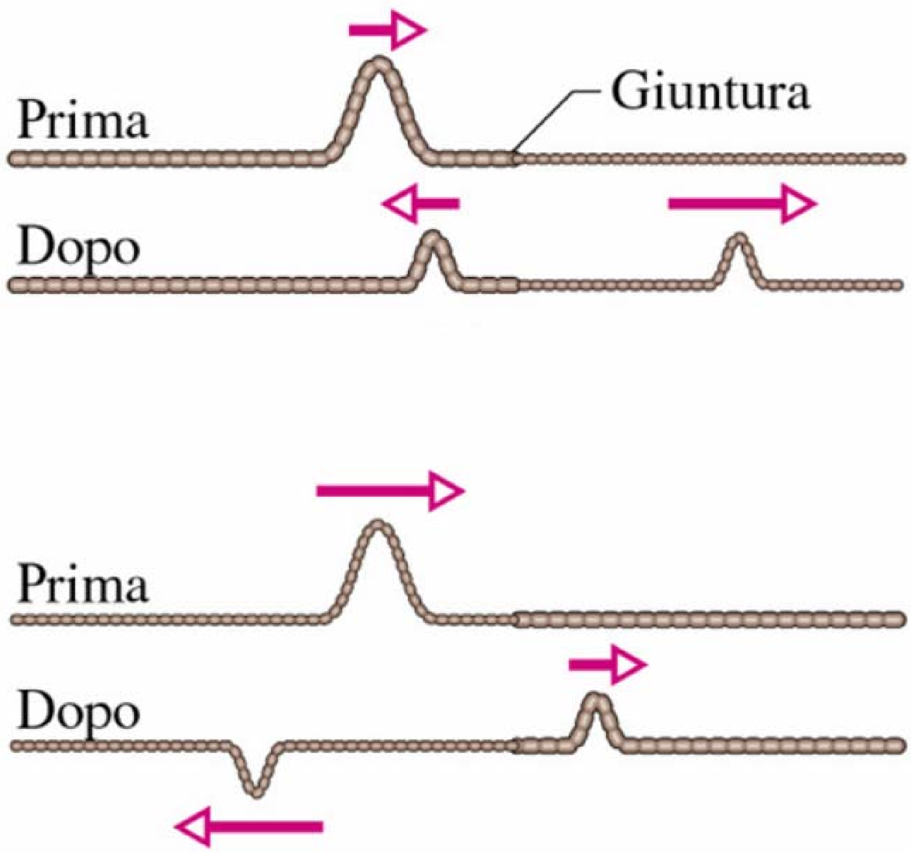


Interferenza

$$y(x,t) = f(x + vt) + f(x - vt)$$

Riflessione e trasmissione



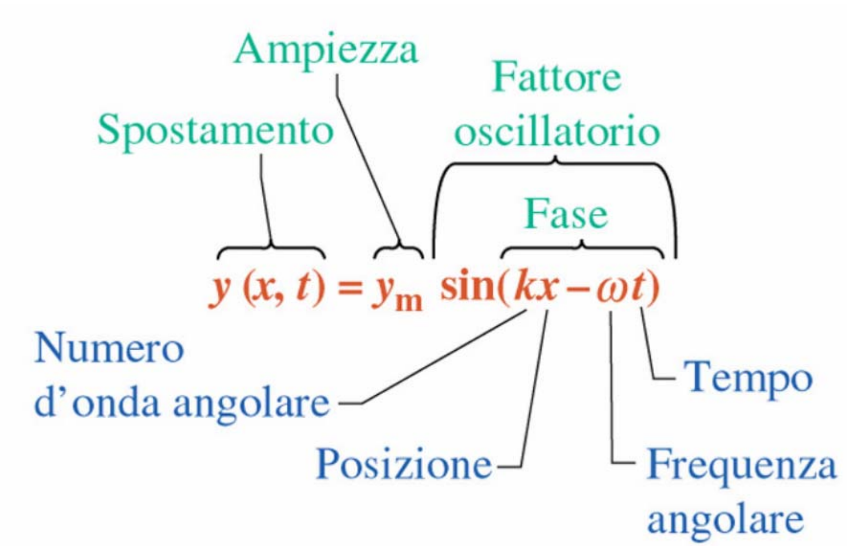


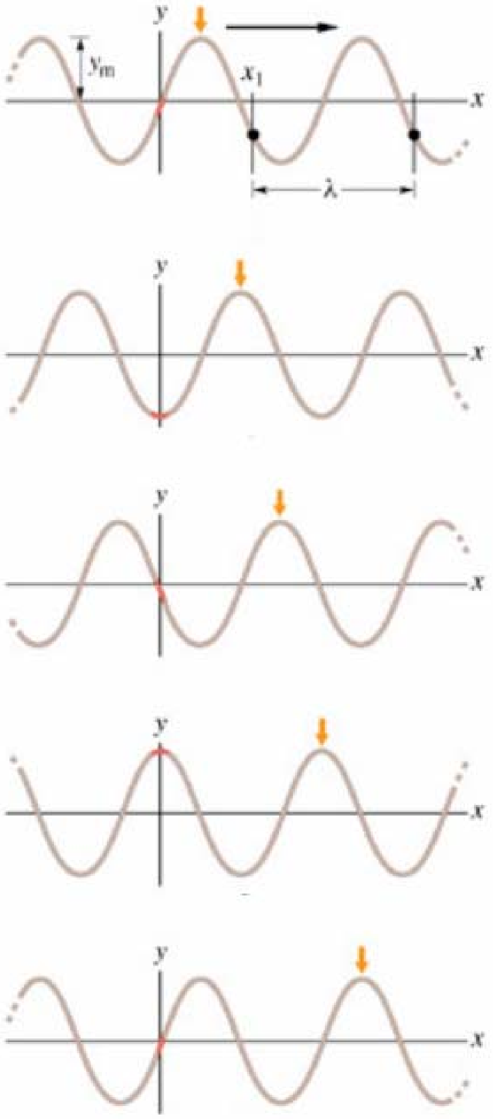
Onde armoniche

$$y = y_m \sin\left(\frac{2\pi}{\lambda}x\right) \qquad x \rightarrow x - vt$$

$$y = y_m \sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$$

$$T = \frac{\lambda}{v} \qquad k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$





Equazione delle onde

$$y(x,t) = A\sin(kx - \omega t)$$

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial t}\left[A\sin(kx - \omega t)\right] = -\omega A\cos(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\frac{\partial y}{\partial t} = \frac{\partial}{\partial t}\left[-\omega A\cos(kx - \omega t)\right] = -\omega^2 A\sin(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = -v^2 k^2 A\sin(kx - \omega t) = -v^2 k^2 y(x,t)$$

Equazione delle onde

$$\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}\left[A\sin(kx - \omega t)\right] = kA\cos(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial y}{\partial x} = \frac{\partial}{\partial x}\left[kA\cos(kx - \omega t)\right] = -k^2 A\sin(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 y(x,t) \quad \Longleftrightarrow \quad \frac{\partial^2 y}{\partial t^2} = -v^2 k^2 y(x,t)$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

Eq. delle onde (lineare)

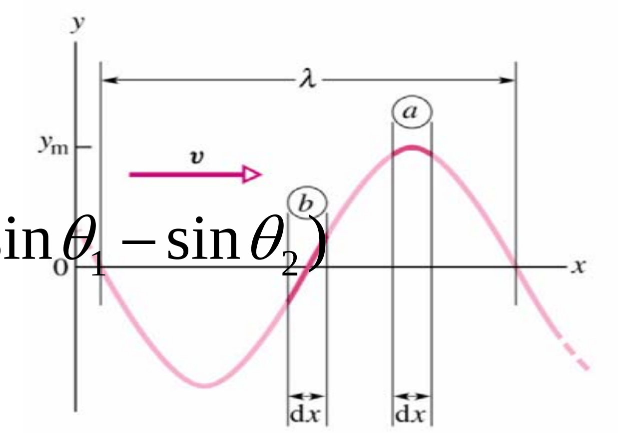
y_1 soluzione

y_2 soluzione

$\Rightarrow$ $y_1 + y_2$ soluzione

Onde su una corda tesa

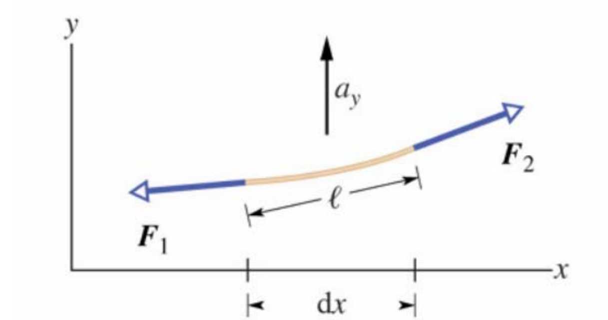
$$|\mathbf{F}_1| = |\mathbf{F}_2| = F$$

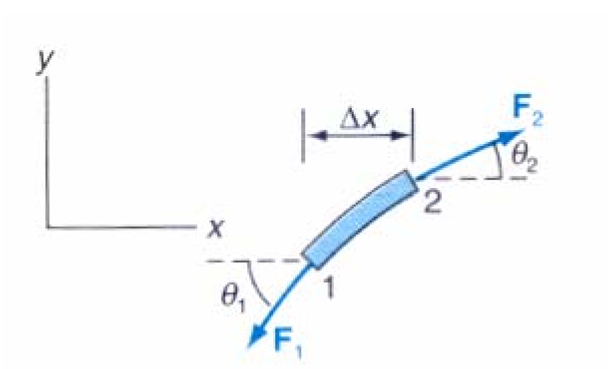


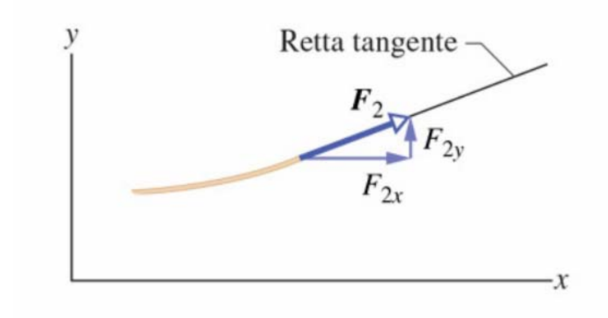
$$\sum F_y = F_{y1} + F_{y2} = -F\sin\theta_1 + F\sin\theta_2 = F(\sin\theta_1 - \sin\theta_2)$$

$$\theta \text{ piccolo} \rightarrow \sin\theta \approx \tan\theta \qquad \tan\theta = \frac{\partial y}{\partial x}$$

$$\sum F_y = F\left[\left(\frac{\partial y}{\partial x}\right)_2 - \left(\frac{\partial y}{\partial x}\right)_1\right]$$







Onde su una corda tesa

$$\left(\frac{\partial y}{\partial x}\right)_2 - \left(\frac{\partial y}{\partial x}\right)_1 = \Delta\frac{\partial y}{\partial x} = \frac{\Delta\frac{\partial y}{\partial x}}{\Delta x}\Delta x = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)\Delta x = \frac{\partial^2 y}{\partial x^2}\Delta x$$

$$\sum F_y = F\frac{\partial^2 y}{\partial x^2}\Delta x \qquad \mu = M/L \text{ densita' lineare}$$

$$m = \mu\Delta x \qquad \sum F_y = ma_y \qquad a_y = \frac{\partial^2 y}{\partial t^2}$$

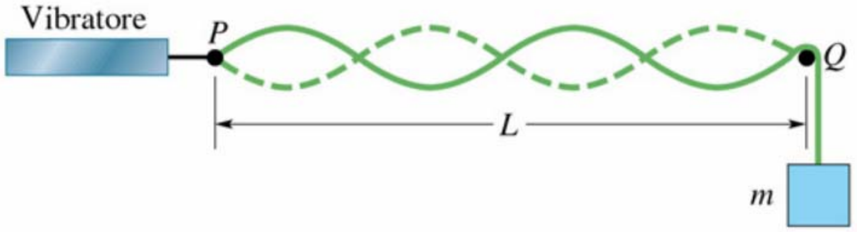
$$F\frac{\partial^2 y}{\partial x^2}\Delta x = \mu\Delta x\frac{\partial^2 y}{\partial t^2} \qquad \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F}\frac{\partial^2 y}{\partial t^2}} \Longleftrightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

$$v = \sqrt{\frac{F}{\mu}} \qquad v = \sqrt{\frac{\text{forza di richiamo}}{\text{massa inerziale}}}$$

Le onde stazionarie

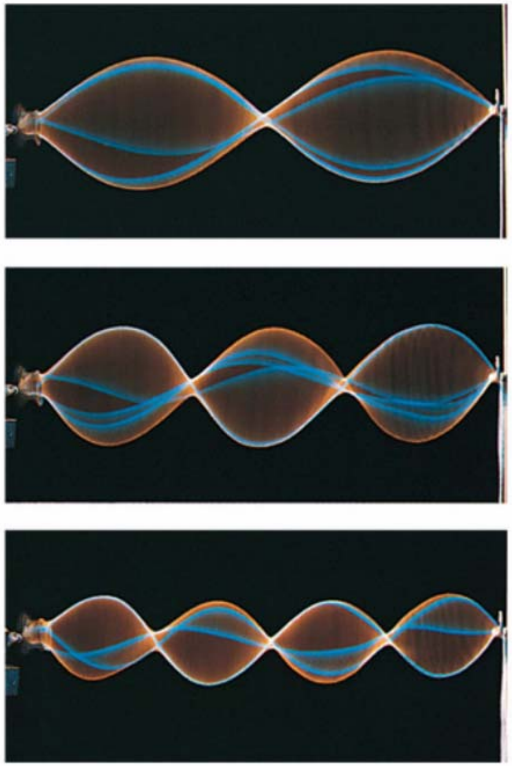
$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$



$$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$\sin\alpha + \sin\beta = 2\sin\left[\frac{\alpha+\beta}{2}\right]\cos\left[\frac{\alpha-\beta}{2}\right]$$



$$\boxed{y(x,t) = 2A\cos(\omega t)\sin(kx)}$$

$$\lambda_n = \frac{2L}{n}$$

$$y(0,t) = 0 \quad y(L,t) = 0 \;\rightarrow$$

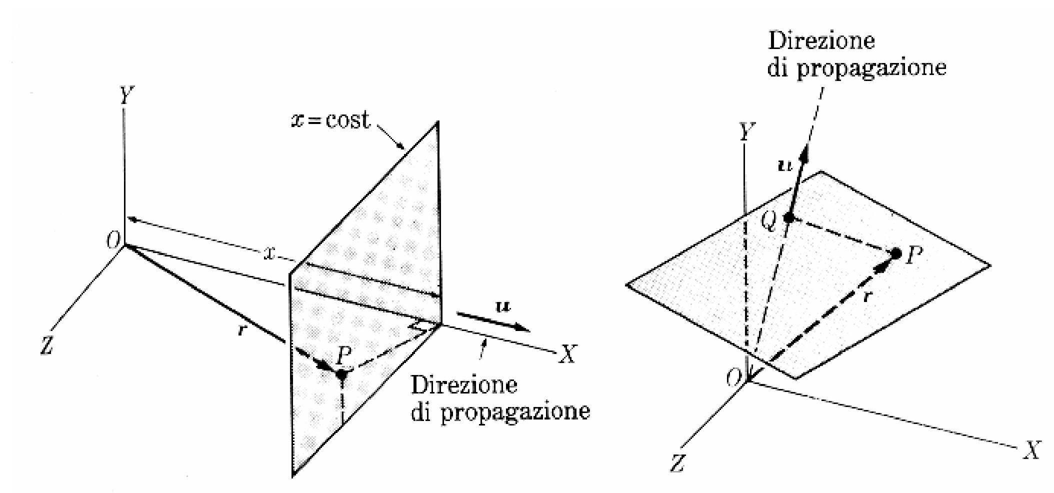
$$\nu_n = n\left(\frac{\sqrt{F/\mu}}{2L}\right)$$

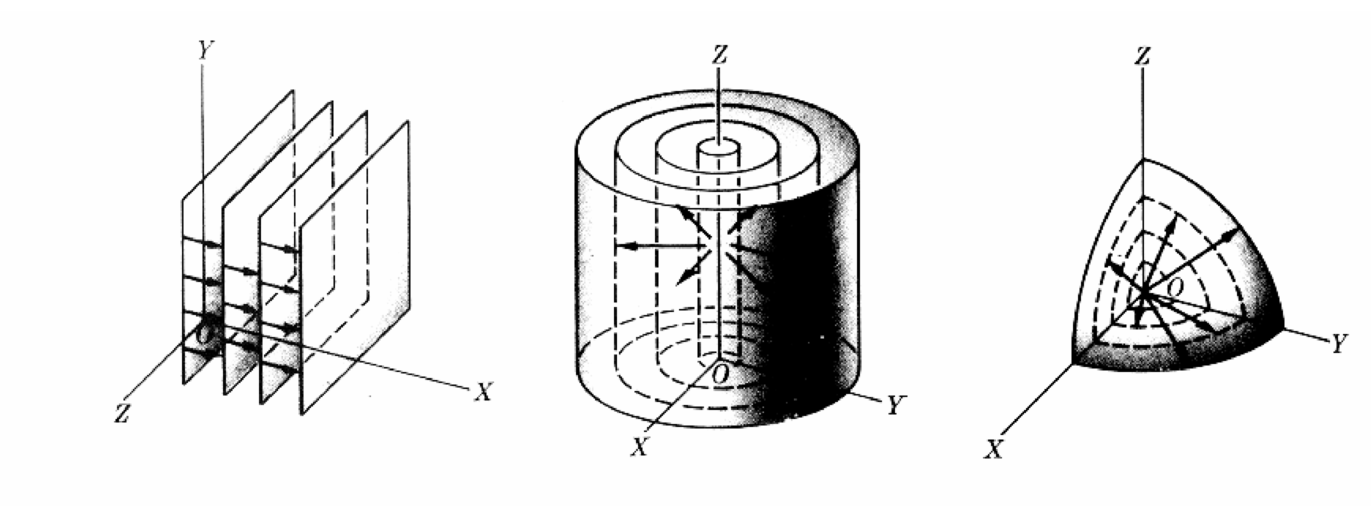
Onde in 2 e 3 dimensioni

$$x = \mathbf{u} \cdot \mathbf{r}$$

$$y(\mathbf{r},t) = A\sin(k(\mathbf{u} \cdot \mathbf{r}) - \omega t)$$

$$y(\mathbf{r},t) = A\sin(\mathbf{k} \cdot \mathbf{r} - \omega t)$$





$$P = \frac{\Delta E}{\Delta t}$$

$$I = \frac{P}{\Delta S} = \frac{\Delta E}{\Delta t \Delta S}$$

$$I = \frac{P_0}{4\pi r^2}$$