Potenziale elettrico

- Energia potenziale
- Potenziale elettrico
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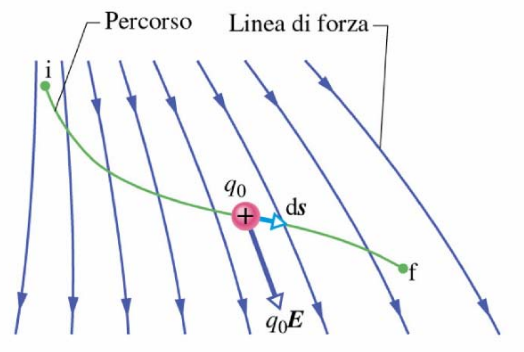
- Relazione tra campo e potenziale
- Proprietà dei conduttori

Energia potenziale elettrica

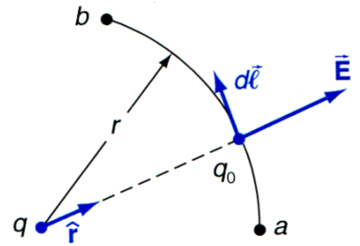
Potenziale elettrico [V]=energia potenziale per unità di carica

$$\int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = q_0 \int_i^f \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$



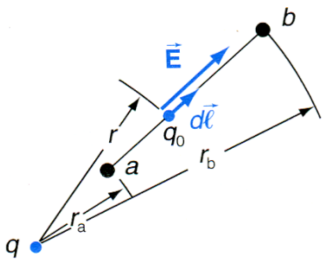
Campo di una carica puntiforme:

$$\int_a^b \vec{\mathbf{F}} \cdot d\vec{\ell} = q_0 \int_a^b \vec{\mathbf{E}} \cdot d\vec{\ell} = 0$$



$$\int_a^b \vec{\mathbf{F}} \cdot d\vec{\ell} = q_0 \int_a^b \vec{\mathbf{E}} \cdot d\vec{\ell} = q_0 \int_{r_a}^{r_b} \frac{q}{4\pi\varepsilon_0 r^2} \hat{\mathbf{r}} \cdot dr\hat{\mathbf{r}} =$$

$$= \frac{q_0 q}{4\pi\varepsilon_0} \int_{r_a}^{r_b} \frac{1}{r^2} dr = \frac{q_0 q}{4\pi\varepsilon_0} \left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

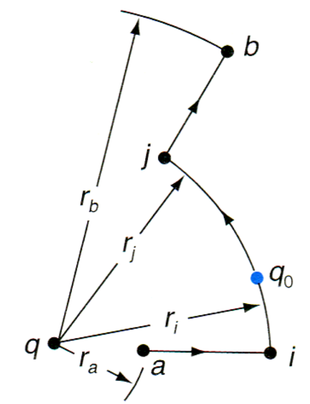


Energia potenziale elettrica

$$\int_a^b \vec{\mathbf{F}} \cdot d\vec{\ell} = \int_a^i \vec{\mathbf{F}} \cdot d\vec{\ell} + \int_i^j \vec{\mathbf{F}} \cdot d\vec{\ell} + \int_j^b \vec{\mathbf{F}} \cdot d\vec{\ell}$$

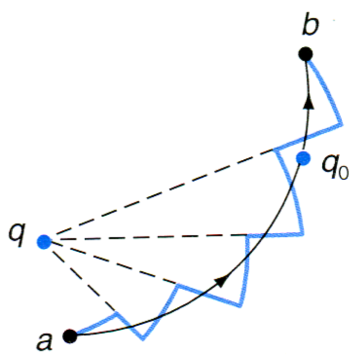
$$= \frac{q_0 q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_i}\right) + 0 + \frac{q_0 q}{4\pi\varepsilon_0}\left(\frac{1}{r_j} - \frac{1}{r_b}\right)$$

$$\int_a^b \vec{\mathbf{F}} \cdot d\vec{\ell} = \frac{q_0 q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$



Anche per un percorso qualsiasi

(campo conservativo)



$$U_b - U_a = -\int_a^b \vec{\mathbf{F}} \cdot d\vec{\ell} = \frac{q_0 q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$U(r) = \frac{q_0 q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)$$

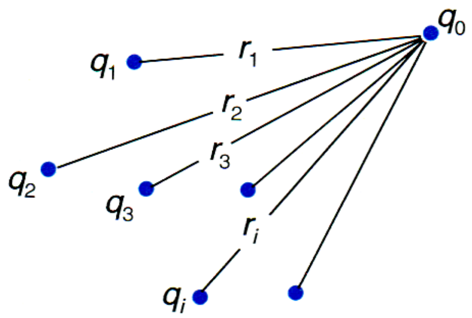
$$U(r) = -\int_\infty^r \vec{\mathbf{F}} \cdot d\vec{\ell}$$

Potenziale=-Lavoro compiuto dalla forza elettrica

Energia potenziale elettrica

Più cariche presenti:

$$\vec{\mathbf{F}} = q_0\vec{\mathbf{E}} = q_0\left(\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\right)$$



$$\int_a^b \vec{\mathbf{F}} \cdot d\vec{\ell} = \int_a^b q_0\left(\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\right) \cdot d\vec{\ell} =$$

$$\int_a^b q_0\vec{\mathbf{E}}_1 \cdot d\vec{\ell} + \int_a^b q_0\vec{\mathbf{E}}_2 \cdot d\vec{\ell}$$

$$U = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$U = \frac{q_0}{4\pi\varepsilon_0}\sum \frac{q_i}{r_i}$$

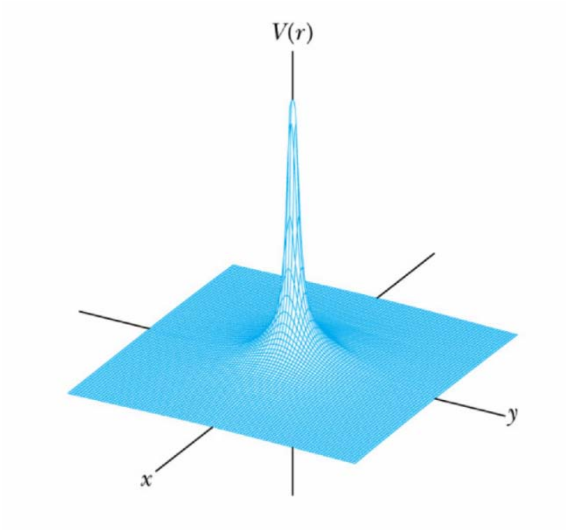
Potenziale elettrico

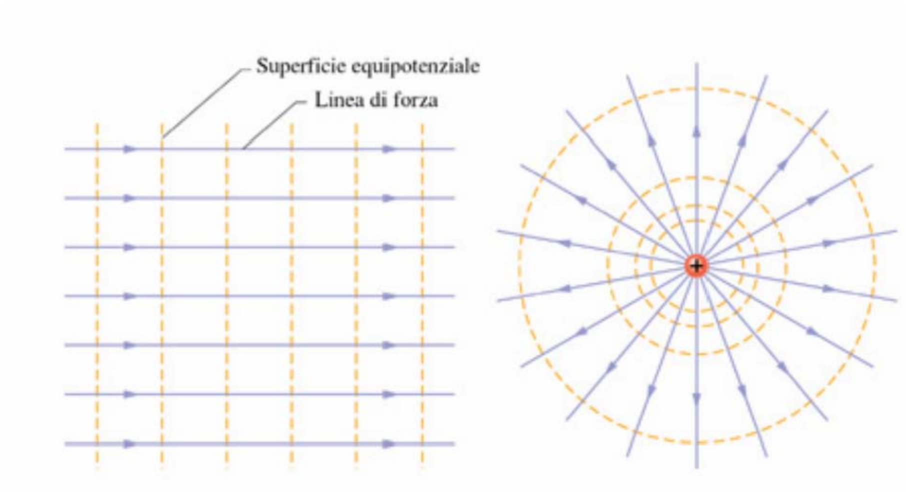
$$V = \frac{U}{q_0}$$ Potenziale elettrico [V]

$$e = 1.6 \cdot 10^{-19} C$$

Particelle cariche: $$V = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{r_i}$$

$$eV = (1.6 \cdot 10^{-19} C)(1V) = \\ = 1.6 \cdot 10^{-19} J$$

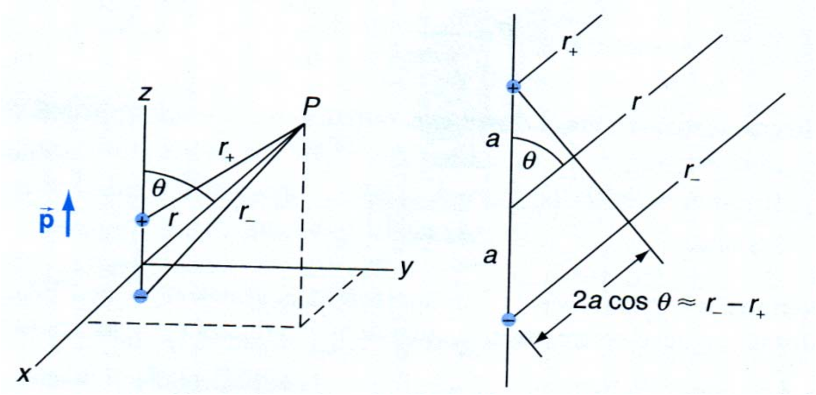




Potenziale elettrico

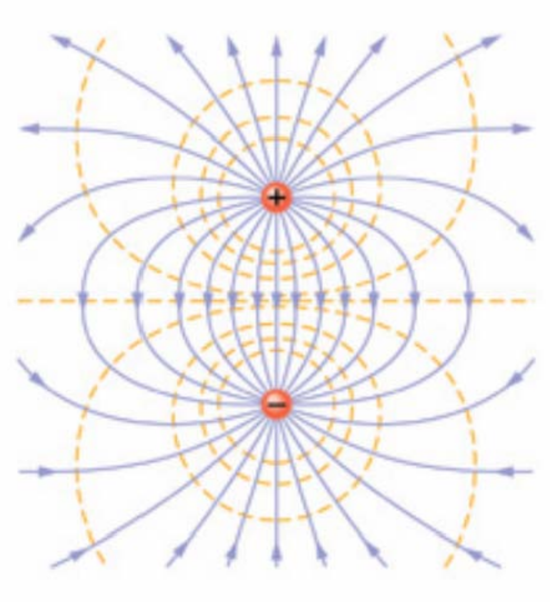
Potenziale del dipolo:

$$V = V_+ + V_- = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$



$$V \approx \frac{2aq\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$V \approx \frac{\vec{\mathbf{p}} \cdot \hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}$$



Potenziale elettrico

Distribuzione continua di carica:

$$V = \frac{1}{4\pi\varepsilon_0} \lim_{N\to\infty, q_i \to 0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\varepsilon_0} \iint_{\substack{Superficie \\ corpo \quad carico}} \frac{dq}{r}$$

Differenza di potenziale

$U = 0 \;\; per \;\; r = \infty$ $\qquad$ $V = 0 \;\; per \;\; r = \infty$

Posizione di riferimento

$$\Delta V = V_b - V_a = \frac{U_b - U_a}{q_0}$$

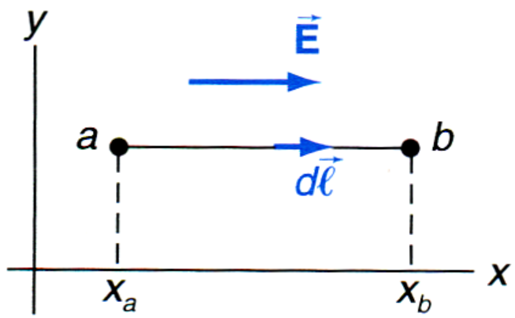
$$U_b - U_a = -q_0 \int_a^b \vec{\mathbf{E}} \cdot d\vec{l}$$

$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{l}$$

Esempio:

$$d\vec{l} = dx\hat{\mathbf{i}}$$

$$V_b - V_a = -\int_{x_a}^{x_b} (E\hat{\mathbf{i}}) \cdot (dx\hat{\mathbf{i}}) = -\int_{x_a}^{x_b} E dx$$



$$V_b - V_a = -E(x_b - x_a)$$

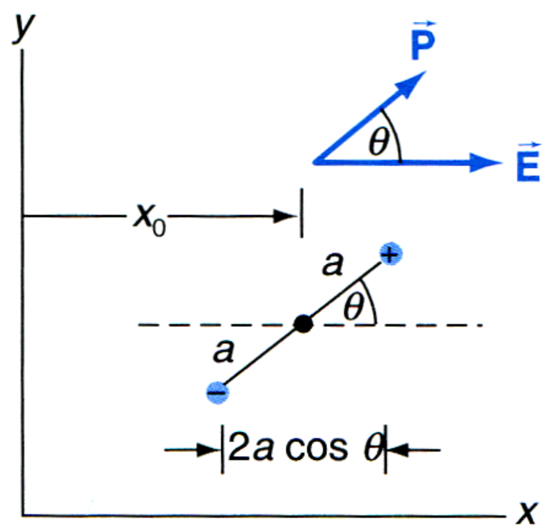
$$V_b - V_a = -E\Delta x$$

Potenziale del dipolo

$$x: \quad x_0 \pm a\cos\theta$$

$$U_+ = q[-E(x_0 + a\cos\theta) + V_0]$$

$$U_- = -q[-E(x_0 - a\cos\theta) + V_0]$$



$$U = U_+ + U_- = q[-E(x_0 + a\cos\theta) + V_0] - q[-E(x_0 - a\cos\theta) + V_0]$$

$$= -2aqE\cos\theta = -pE\cos\theta$$

$$U = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$$

Relazione tra campo e potenziale

$$V = -\int_{\infty}^{P} \vec{\mathbf{E}} \cdot d\vec{l}$$

$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{l} \qquad a = (x, y, z) \;\; b = (x + \Delta x, y, z)$$

$$\vec{\mathbf{E}} \cdot d\vec{l} = (E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}}) \cdot (dx' \hat{\mathbf{i}}) = E_x dx'$$

$$V(x + \Delta x, y, z) - V(x, y, z) = -\int_x^{x+\Delta x} E_x dx'$$

$$\lim_{\Delta x \to 0} \Rightarrow -E_x \int_x^{x+\Delta x} dx' = -E_x [(x + \Delta x) - (x)] = -E_x \Delta x$$

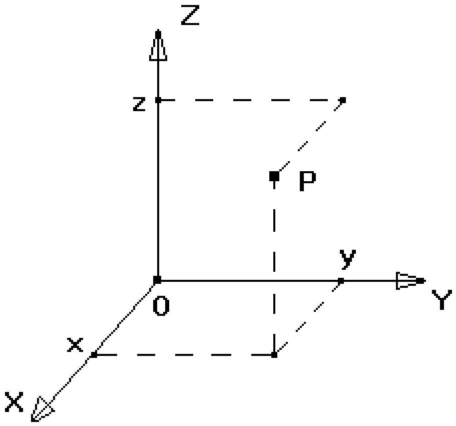
$$V(x + \Delta x, y, z) - V(x, y, z) \approx -E_x \Delta x$$

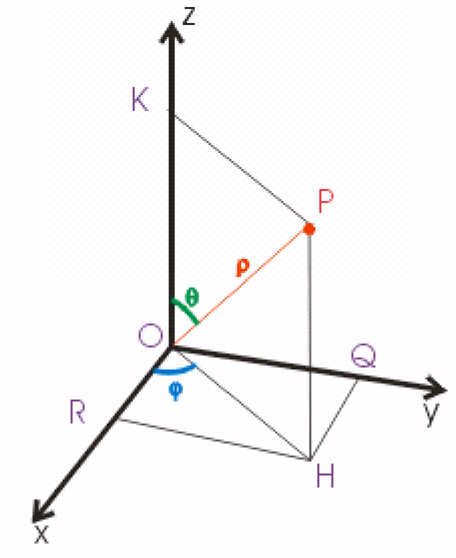
$$\lim_{\Delta x \to 0} \left(\frac{V(x + \Delta x, y, z) - V(x, y, z)}{\Delta x} \right) = -E_x \Rightarrow E_x = -\frac{\partial V}{\partial x}$$

Relazione tra campo e potenziale

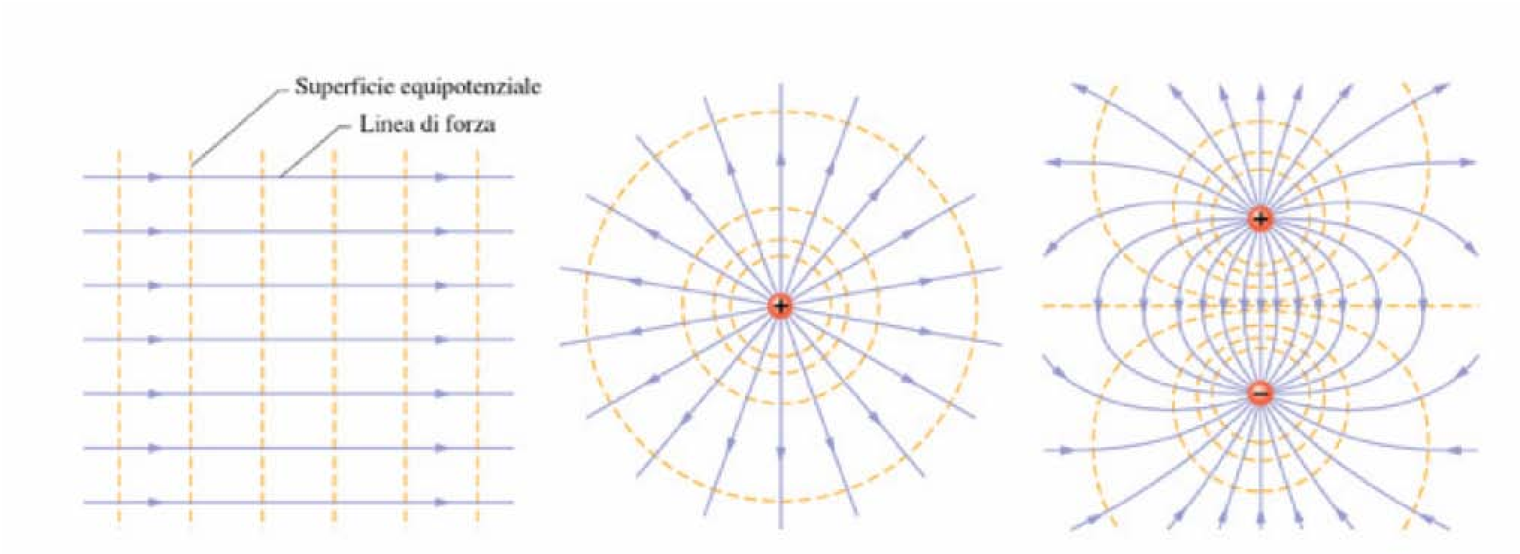
$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

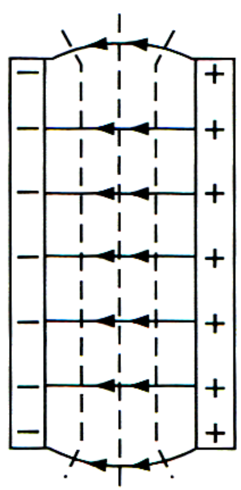
$$\vec{\mathbf{E}} = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right) \qquad \text{opp.} \qquad E_r = -\left(\frac{\partial V}{\partial r}\right)$$

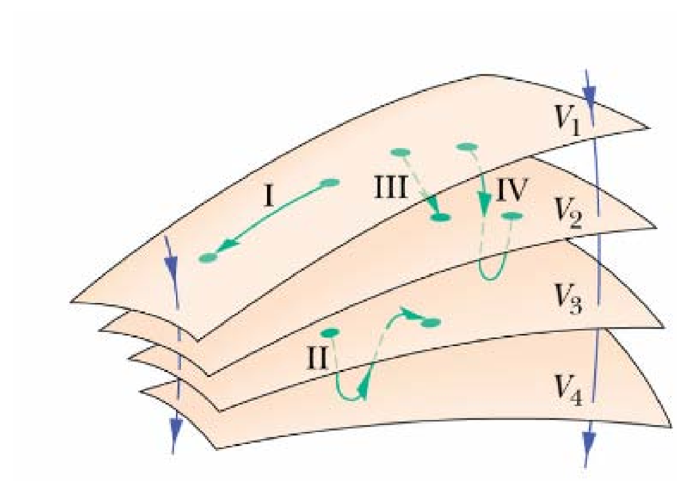




Superfici equipotenziali

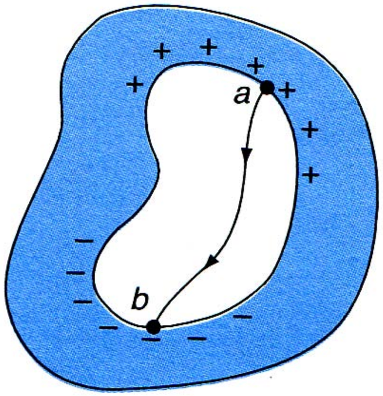






Proprietà dei conduttori

$$V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{l}$$



Generatore Van der Graaf