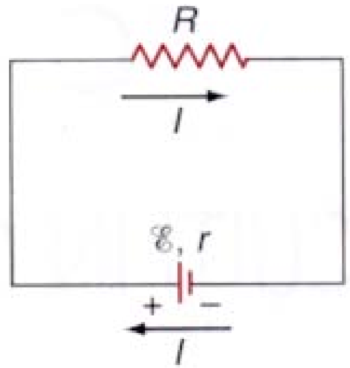
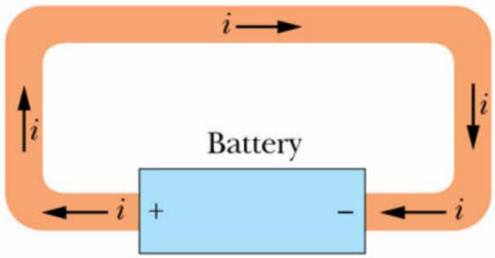
Circuiti in corrente continua

- f.e.m. di una batteria
- Energia elettrica e potenza
- Leggi di Kirchhoff
- Circuiti RC

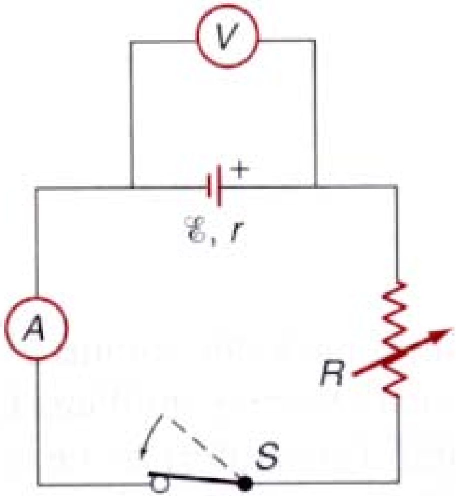
Batteria

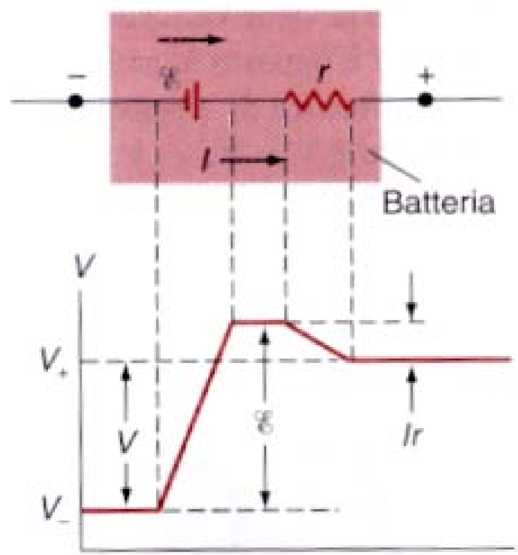




$$f.e.m. = \text{lavoro per unità di carica}$$

$$[V] = \left[\frac{J}{C}\right]$$



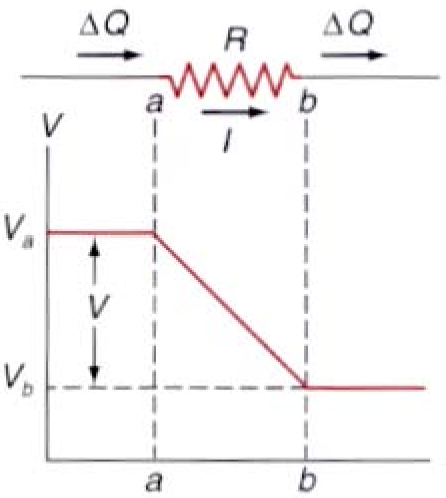


$$r = \text{resistenza interna}$$

$$V = \varepsilon - Ir$$

Circuito chiuso

Energia elettrica



$$\Delta U = V_b \Delta Q - V_a \Delta Q = -V \Delta Q$$

$$P_R = -\frac{\Delta U}{\Delta t} = -\frac{-V \Delta Q}{\Delta t} = V \frac{\Delta Q}{\Delta t}$$

Potenza dissipata dalla resistenza

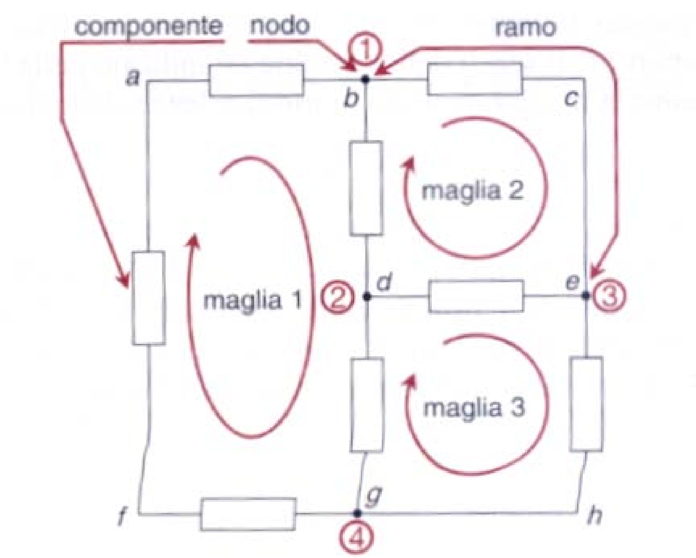
$$P_R = VI \quad = I^2 R = \frac{V^2}{R}$$

Legge di Joule

Batteria

$$P_u = \frac{\Delta U}{\Delta t} = \frac{V \Delta Q}{\Delta t} = VI \quad = \varepsilon I - I^2 r$$

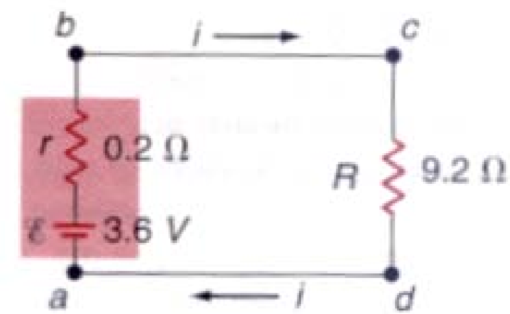
Leggi di Kirchhoff



$$\sum_i V_i = 0$$

Legge delle maglie

Esempio:



$$(V_b - V_a) + (V_c - V_b) + (V_d - V_c) + (V_a - V_d) = 0$$

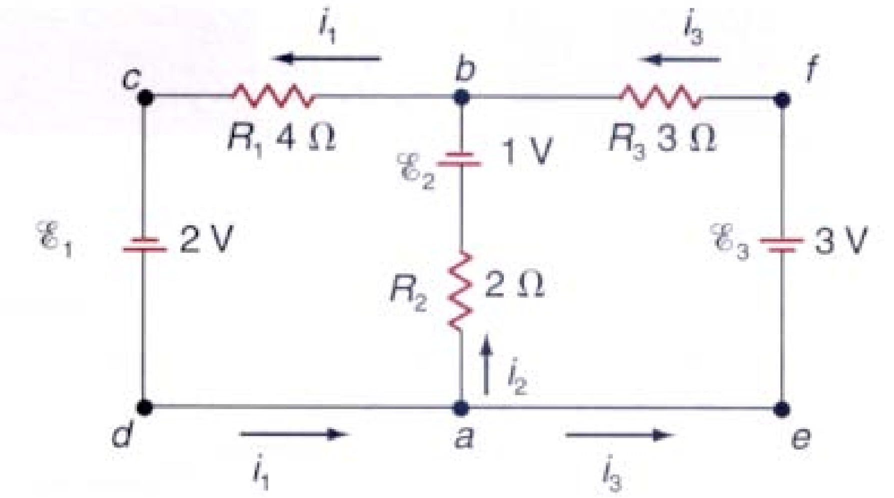
$$(\varepsilon - ir) + (0) + (-iR) + (0) = 0$$

$$i = \frac{\varepsilon}{r + R}$$

Leggi di Kirchhoff

$$\sum i_{entranti} = \sum i_{uscenti}$$

Legge dei nodi



Esempio:

$$i_1 = i_2 + i_3$$

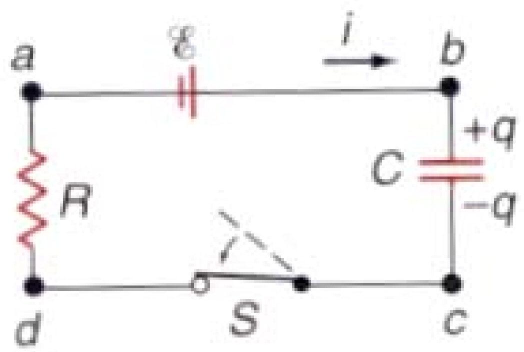
$$\left(-i_2 R_2\right) + \left(-\varepsilon_2\right) + \left(-i_1 R_1\right) + \left(\varepsilon_1\right) = 0$$ Maglia di sn.

$$\left(\varepsilon_3\right) + \left(-i_3 R_3\right) + \left(\varepsilon_2\right) + \left(i_2 R_2\right) = 0$$ Maglia di ds.

3 eq. 3 incognite → $i_1 = 0.5\text{A},\ i_2 = -0.5\text{A},\ i_3 = 1\text{A}$

Circuiti RC

Carica del condensatore



$$q = CV$$

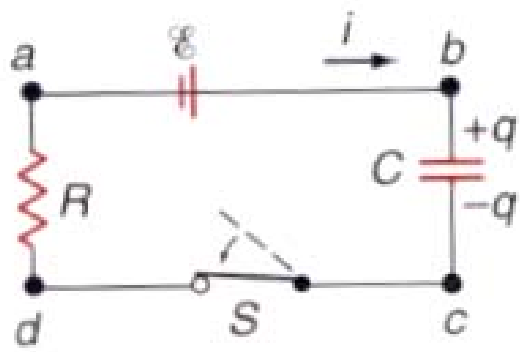
$$i = \frac{dq}{dt} \leftarrow \text{Carica sul condensatore}$$

$$(V_b - V_a) + (V_c - V_b) + (V_d - V_c) + (V_a - V_d) = 0$$

$$(\varepsilon) + \left(\frac{-q}{C}\right) + (0) + (-iR) = 0$$

$$\frac{-dq}{\varepsilon C - q} = \frac{1}{RC}dt \qquad \int_0^q \frac{-dq'}{\varepsilon C - q'} = \int_0^t \frac{1}{RC}dt'$$

Circuiti RC

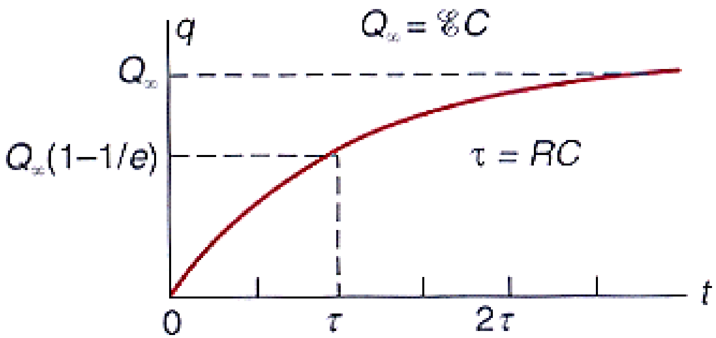


$$\ln(\varepsilon C - q) = -\frac{1}{RC} + \ln(\varepsilon C)$$

$$\ln\left(\frac{\varepsilon C - q}{\varepsilon C}\right) = -\frac{t}{RC}$$

$$\frac{\varepsilon C - q}{\varepsilon C} = e^{-\frac{t}{RC}}$$

$$q(t) = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right)$$



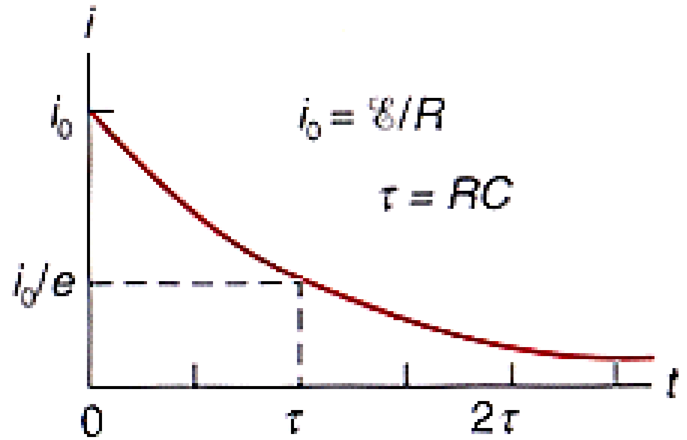
$\tau = RC$ costante di tempo

Circuiti RC

$$i = \frac{dq}{dt} \qquad q(t) = \varepsilon C\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i = \varepsilon C\left(-\frac{1}{RC}\right)\left(-e^{-\frac{t}{RC}}\right)$$

$$i = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} = i_0 e^{-\frac{t}{\tau}}$$



Circuiti RC

Energia fornita dalla batteria

$$\varepsilon Q_\infty = \varepsilon(\varepsilon C) = \varepsilon^2 C$$

Energia del condensatore

$$\frac{1}{2}\frac{Q_\infty^{\,2}}{C} = \frac{1}{2}\varepsilon^2 C$$

Potenza dissipata dalla resistenza

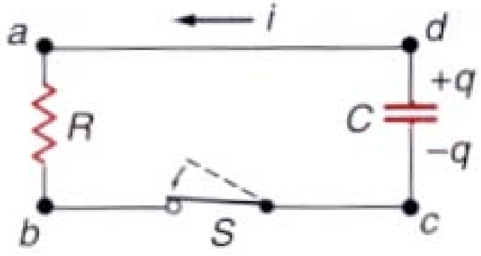
$$P = i^2 R = -\frac{dU}{dt}$$

$$-\Delta U = \int_0^\infty i^2 R dt = \int_0^\infty \left(\frac{\varepsilon}{R} e^{-t/RC}\right)^2 R dt = \left(\frac{\varepsilon}{R}\right)^2 \left(\frac{RC}{2}\right) R \int_0^\infty e^{-x} dx$$

$$-\Delta U = \frac{1}{2}\varepsilon^2 C$$

Circuiti RC

Scarica del condensatore



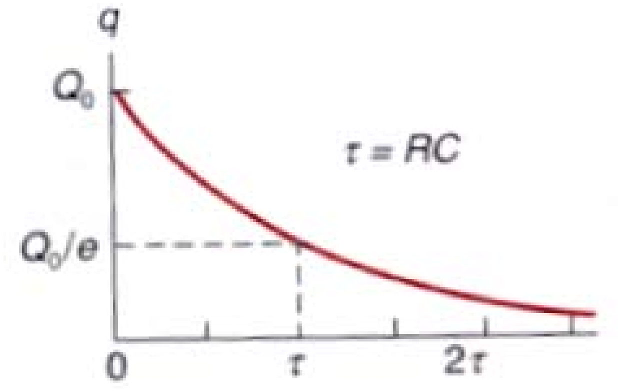
$$i = -\frac{dq}{dt} \qquad -iR + \frac{q}{C} = 0$$

$$-\frac{dq}{dt}R = \frac{q}{C} \qquad \frac{dq}{q} = -\frac{1}{RC}dt \qquad \int_{Q_0}^{q} \frac{dq'}{q'} = -\int_{0}^{t} \frac{1}{RC}dt'$$

$$\ln\left(\frac{q}{Q_0}\right) = -\left(\frac{t}{RC}\right) \qquad q(t) = Q_0 e^{-t/\tau}$$

Circuiti RC

$$q(t) = Q_0 e^{-t/\tau}$$



$$i = -\frac{d}{dt}\left(Q_0 e^{-t/RC}\right)$$

$$i = \frac{Q_0}{RC} e^{-t/RC}$$

$$i(t) = \frac{V_0}{R} e^{-t/RC} = i_0 e^{-t/\tau}$$

$$V_0 = \frac{Q_0}{C}$$